Dot Product of Vectors 3-11-08

Dot Product (or inner product):

$$\vec{u} = \langle u_{1,} u_2 \rangle \qquad \vec{v} = \langle v_{1,} v_2 \rangle$$

$$\vec{u} \bullet \vec{v} =$$

1. Find the dot product of $\vec{u}$ and $\vec{v}$.

$\vec{u} = \langle -2, 7 \rangle, \quad \vec{v} = \langle -5, -8 \rangle$

$\vec{u} = 4\hat{i} - 11\hat{j}, \quad \vec{v} = -3\hat{j}$

Properties of the Dot Product

1. $\vec{u} \bullet \vec{v} =$

2. $\vec{u} \bullet \vec{u} =$

3. $0 \bullet \vec{u} =$

4. $\vec{u} \bullet \left(\vec{v} + \vec{w} \right) =$

5. $\left(c\vec{u} \right) \bullet \vec{v} = \vec{u} \bullet \left(c\vec{v} \right) =$

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2. Use the dot product to find $|\vec{u}|$.

   $\vec{u} = \langle -7, 2 \rangle$ $\vec{u} = 13\hat{i} - 4\hat{j}$

Angle Between Two Vectors

If θ is the angle between the nonzero vectors $\vec{u}$ and $\vec{v}$, then

$$\cos\theta = \frac{\vec{u} \bullet \vec{v}}{|\vec{u}||\vec{v}|}$$

3. Find the angle θ between the following vectors.

   $\vec{u} = \langle 5, 2 \rangle$ $\vec{v} = \langle -6, -1 \rangle$

4. Find the angle θ between the following vectors.

   $\vec{u} = \left(\cos\frac{\pi}{3}\right)\hat{i} + \left(\sin\frac{\pi}{3}\right)\hat{j}, \quad \vec{v} = \left(3\cos\frac{5\pi}{6}\right)\hat{i} + \left(3\sin\frac{5\pi}{6}\right)\hat{j}$

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Orthogonal Vectors

The vectors $\vec{u}$ and $\vec{v}$ are orthogonal( ) if and only if

$$\vec{u} \bullet \vec{v} = \quad .$$

5. Prove that the vectors $\vec{u}$ and $\vec{v}$ are orthogonal.

$$\vec{u} = \langle -4, -1 \rangle \qquad \vec{v} = \langle 1, -4 \rangle$$

Projection of $\vec{u}$ onto $\vec{v}$

If $\vec{u}$ and $\vec{v}$ are nonzero vectors, the projection of $\vec{u}$ onto $\vec{v}$ is

$$proj_v u = \left(\frac{\vec{u} \bullet \vec{v}}{\left|\vec{v}\right|^2} \right) \vec{v}$$

6. Find the vector projection of $\vec{u}$ onto $\vec{v}$.

$$\vec{u} = \langle -2, 8 \rangle \qquad \vec{v} = \langle 9, -3 \rangle$$