

**Extra Credit Due Today!!!!!!!!!!!!****The Natural Base “e”**

$$e = \lim_{x \rightarrow \infty} \left( 1 + \frac{1}{x} \right)^x$$

$$f(x) = a \cdot b^x \quad \Rightarrow \quad f(x) = a \cdot e^{kx}$$

where  $b^x = e^{kx}$

**The Exponential Function**

$$f(x) = a \cdot e^{kx}$$

1. Describe the transformation of the function  $f(x) = e^x$ .

a.  $f(x) = e^{3x}$

b.  $f(x) = e^{-x}$

c.  $f(x) = 3e^x$

What are some examples of natural phenomena where exponential growth is a reasonable model?

**Logistic Functions (Restricted Growth)**

$$f(x) = \frac{c}{1 + a \cdot b^x} \quad \text{or} \quad f(x) = \frac{c}{1 + a \cdot e^{-kx}}$$

where **a**, **b**, **c**, and **k** are positive constants

**c** is the limit of growth or decay

If  $b < 1$  or  $k > 0$ , then logistic growth.

If  $b > 1$  or  $k < 0$ , then logistic decay.

**The Logistic Function**

$$f(x) = \frac{1}{1 + e^{-x}}$$

2. Find the y-intercept and the horizontal asymptotes.

$$f(x) = \frac{7}{1 + 2.5 \cdot 0.4^x}$$

3. Find the y-intercept and the horizontal asymptotes.

$$f(x) = \frac{14}{1 + 6 \cdot e^{2x}}$$

**Population Models**

4. In 1990 the population of the city of Spokane was 177,196 and 195,629 in 2000.

Assuming exponential growth, what will be the city's population in the year 2020?

When will the city grow to 260,000 people?

5. Based on population data from 1980 to 2000, a logistic model for the population of Seattle  $t$  years after 1980 is

$$P(t) = \frac{3338230}{1 + 1.077e^{-0.05177t}} .$$

According to this model, when was the population 2 million?

When will the population reach 3 million?