Calculus Chap. 7 section 4 Length of curves	Name: ____________________ Date: ____________________ Period: ____________________

1. Do the following for the curve $y = 5x^2 - 6, \quad -2 \le x \le 2$.

a. Set up an integral for the length of the curve.
b. Graph the curve to see what it looks like.
c. Use your grapher's or computer's integral evaluator to find the curve's length numerically.

If f is continuously differentiable on the closed interval [a,b], the length of the curve (graph) y = f(x) from x = a to x = b is the following.

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

1. answer

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If f is continuously differentiable on the closed interval [a,b], the length of the curve (graph) $y = f(x)$ from $x = a$ to $x = b$ is the following.

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

a. To set up an integral for the length of the curve, use the formula from above with $a = -2$, $b = 2$, and $y = 5x^2 - 6$. Begin by finding $\frac{dy}{dx}$.

$$\frac{dy}{dx} = 10x$$

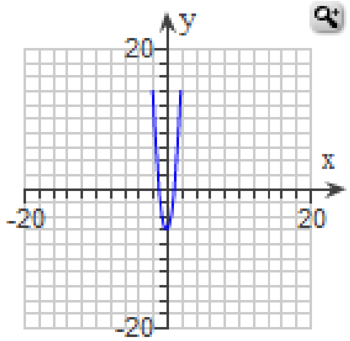
Substitute the result in the formula and simplify.

$$\begin{aligned} L &= \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \\ &= \int_{-2}^{2} \sqrt{1 + (10x)^2}\, dx \\ &= \int_{-2}^{2} \sqrt{1 + 100x^2}\, dx \end{aligned}$$

Thus, the integral for the length of the curve is $\int_{-2}^{2} \sqrt{1 + 100x^2}\, dx$.

1. answer cont.

b. Use a grapher to graph the curve $y = 5x^2 - 6$ on the interval $-2 \le x \le 2$. The graph is shown below.



c. To find the curve's length numerically, use your grapher's or computer's integral evaluator to evaluate the integral for the length of the curve.

$$L = \int_{-2}^{2} \sqrt{1 + 100x^2}\, dx \approx 40.42$$

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2. Use a grapher to find the curve's length numerically.

$x = 2\ \mathbf{sin}\ y,\ \frac{\pi}{6} \le y \le \frac{5\pi}{6}$

The length L of a curve on $a \le y \le b$ defined by $x = g(y)$ is $L = \int_a^b \sqrt{\left[1 + \left(\frac{dx}{dy}\right)^2\right]}\, dy$.

2. answer

Use a grapher to find the curve's length numerically.

$x = 2 \sin y, \ \frac{\pi}{6} \le y \le \frac{5\pi}{6}$

The length L of a curve on $a \le y \le b$ defined by $x = g(y)$ is $L = \int_a^b \sqrt{\left[1 + \left(\frac{dx}{dy}\right)^2\right]}\, dy$.

$x = 2 \sin y$

$\frac{dx}{dy} = 2 \cos y$

So, $L = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sqrt{1 + 4 \cos^2 y}\, dy$.

Using a grapher to evaluate the integral, the curve length is 3.01 rounded to the nearest hundredth.

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3.

For $x = \sqrt{144 - y^2}$, $-6 \le y \le 6$, **(a)** set up an integral for the length of the curve; **(b)** graph the curve to see what it looks like; and **(c)** use NINT or a similar numerical integration feature on your calculator to find the length of the curve.

(a) To set up an integral for the length of the curve, first decide which arc length formula to use.

3. answer

For $x = \sqrt{144 - y^2}$, $-6 \le y \le 6$, **(a)** set up an integral for the length of the curve; **(b)** graph the curve to see what it looks like; and **(c)** use NINT or a similar numerical integration feature on your calculator to find the length of the curve.

(a) To set up an integral for the length of the curve, first decide which arc length formula to use.

If a smooth curve begins at (a,c) and ends at (b,d), a < b, c < d, then the length (arc length) of the curve is given by the formulas shown below.

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \quad \text{if y is a smooth function of x on [a,b]}$$

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \quad \text{if x is a smooth function of y on [c,d]}$$

Since the given function is solved for x, use the formula $L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$.

Now take the derivative of $x = \sqrt{144 - y^2}$ with respect to y.

$$\frac{dx}{dy} = -\frac{y}{\sqrt{144 - y^2}}$$

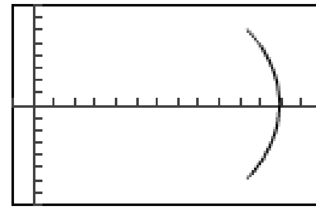
Substitute the value of $\frac{dx}{dy}$ into the formula $L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$ and simplify. The integral over the interval $-6 \le y \le 6$ is shown below.

$$L = \int_{-6}^{6} \sqrt{1 + \frac{y^2}{144 - y^2}}\, dy$$

(b) To graph the curve, solve the function $x = \sqrt{144 - y^2}$ for y and graph over the interval $-6 \le y \le 6$.

3. answer cont.

The correct graph of $x = \sqrt{144 - y^2}$ over the interval $-6 \le y \le 6$ is shown to the right on the viewing window $[-1, 14, 1]$ by $[-8, 8, 1]$.



(c) Use NINT or a similar numerical integration feature on your calculator to find the length of the curve. Recall that the calculator approximation for the definite integral $\int_a^b f(x)dx$ is $\text{NINT}(f(x), x, a, b)$.

Approximate $L = \int_{-6}^{6} \sqrt{1 + \frac{y^2}{144 - y^2}}\, dy.$

$$L \approx \text{NINT}\left(\sqrt{1 + \frac{y^2}{144 - y^2}}, y, -6, 6\right) \approx 12.566$$

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4.

Use a grapher or computer to find the length of the curve numerically.

$3y^2 + 3y = x + 5$ from $(-5, -1)$ to $(31, 3)$

The length, L, of a curve on $a \le y \le b$ defined by $x = g(y)$ is: $L = \int_a^b \sqrt{\left[1 + \left(\frac{dx}{dy}\right)^2\right]}\, dy$.

4. answer

Use a grapher or computer to find the length of the curve numerically.

$3y^2 + 3y = x + 5$ from $(-5, -1)$ to $(31, 3)$

The length, L, of a curve on $a \le y \le b$ defined by $x = g(y)$ is: $L = \int_a^b \sqrt{\left[1 + \left(\frac{dx}{dy}\right)^2\right]}\, dy$.

$3y^2 + 3y = x + 5$. $\frac{dx}{dy} = 6y + 3$.

So, $L = \int_{-1}^{3} \sqrt{1 + (6y + 3)^2}\, dy$.

Thus, the curve length is 38.05 rounded to the nearest hundredth.

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5.

Find the length of the following curve. If you have a grapher, you may want to graph the curve to see what it looks like.

$$y = \frac{1}{3}(x^2 + 2)^{3/2} \quad \text{from } x = 0 \text{ to } x = 9$$

If f is continuously differentiable on the closed interval [a,b], the length of the curve (graph) $y = f(x)$ from $x = a$ to $x = b$ is the following.

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_a^b \sqrt{1 + [f\,'(x)]^2}\, dx$$

5. answer

Find the length of the following curve. If you have a grapher, you may want to graph the curve to see what it looks like.

$$y = \frac{1}{3}(x^2+2)^{3/2} \quad \text{from } x = 0 \text{ to } x = 9$$

If f is continuously differentiable on the closed interval [a,b], the length of the curve (graph) $y = f(x)$ from $x = a$ to $x = b$ is the following.

$$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$$

To determine the length of the given curve use the formula from above with $a = 0$, $b = 9$, and $y = \frac{1}{3}(x^2+2)^{3/2}$. Begin by finding $\frac{dy}{dx}$.

$$\frac{dy}{dx} = x(x^2+2)^{1/2}$$

Since $\frac{dy}{dx}$ is squared in the formula, square the result from above and simplify.

$$\left(\frac{dy}{dx}\right)^2 = [x(x^2+2)^{1/2}]^2$$

$$= x^2(x^2+2)$$

$$= x^4+2x^2 \qquad \text{Multiply.}$$

Now substitute the expression for $\left(\frac{dy}{dx}\right)^2$ into the formula for length along with the limits of the integral.

$$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_0^9 \sqrt{1+x^4+2x^2}\,dx$$

5. answer cont.

Rearrange the terms of the expression in the radicand. Notice that $1+2x^2+x^4$ is a perfect square trinomial. In order to simplify further, rewrite the expression as a perfect square.

$$\int_0^9 \sqrt{1+x^4+2x^2}\,dx = \int_0^9 \sqrt{(1+x^2)^2}\,dx$$

Simplify and evaluate the integral. Find the antiderivative of $1+x^2$.

$$\int_0^9 \sqrt{(1+x^2)^2}\,dx = \int_0^9 (1+x^2)dx$$

$$= \left[x+\frac{x^3}{3}\right]_0^9$$

$$\left[x+\frac{x^3}{3}\right]_0^9 = \left(9+\frac{9^3}{3}\right)-\left(0+\frac{0^3}{3}\right)$$

$$= 252$$

Thus, the length of the curve $y=\frac{1}{3}(x^2+2)^{3/2}$ from $x=0$ to $x=9$ is 252. The graph of the curve is shown to the right.