Calculus	Name: ______________________
Chap. 7 Applications of integrals	Date: ______________________
Visualize the shape practice	Period: ______________________

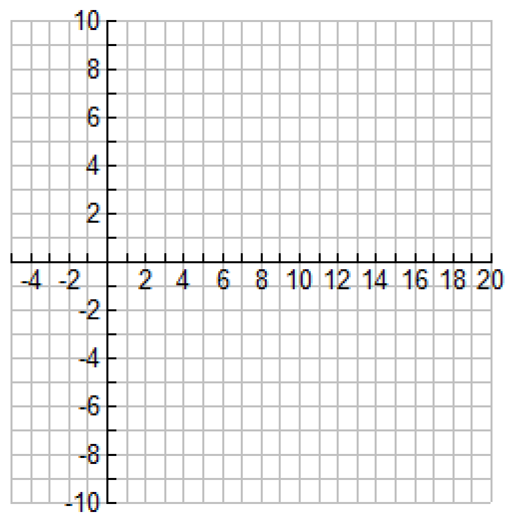
In each of problems below:

A. Sketch the shape on the graph

B. Write the formula (equation) to calculate the area described

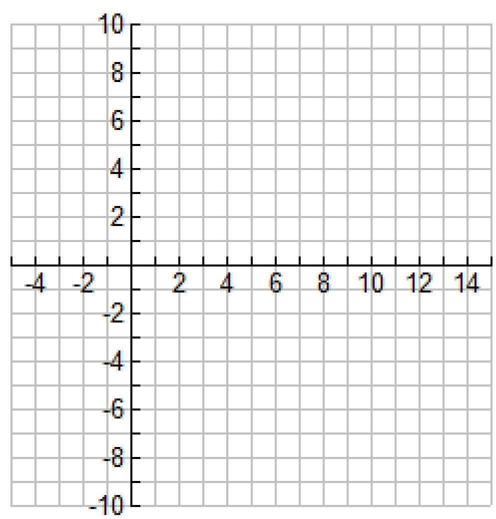
1. A solid lies between planes perpendicular to the x-axis at $x = 0$ and $x = 14$. The cross-sections perpendicular to the axis on the interval $0 \le x \le 14$ are squares with diagonals that run from the parabola $y = -2\sqrt{x}$ to the parabola $y = 2\sqrt{x}$. Find the volume of the solid.

First find the length of the segment, $L = f(x)$, joining opposite points on the top and bottom parabolas.



2. Find the volume of the solid generated by revolving the function about the x-axis (Note: the left side of the shape is x=0)

$$3x + 5y = 30$$



Answers

1. A solid lies between planes perpendicular to the x-axis at $x = 0$ and $x = 14$. The cross-sections perpendicular to the axis on the interval $0 \le x \le 14$ are squares with diagonals that run from the parabola $y = -2\sqrt{x}$ to the parabola $y = 2\sqrt{x}$. Find the volume of the solid.

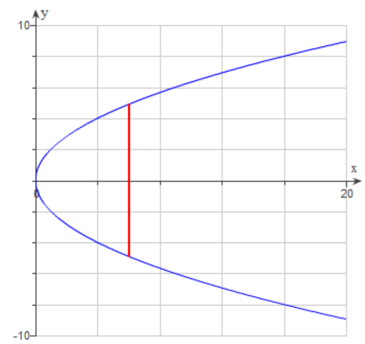
First find the length of the segment, $L = f(x)$, joining opposite points on the top and bottom parabolas.

$L = f(x) = 4\sqrt{x}$

The area of the square in terms of its diagonal, D, is $\frac{D^2}{2}$.

So the area, A, of the square cross-section in terms of x is $A(x) = \frac{D^2}{2} = \frac{(4\sqrt{x})^2}{2} = 8x$.

The volume, V, of a solid of known integrable cross-sectional area A(x) from $x = a$ to $x = b$ is the integral of A from a to b. Symbolically: $V = \int_a^b A(x)dx$.



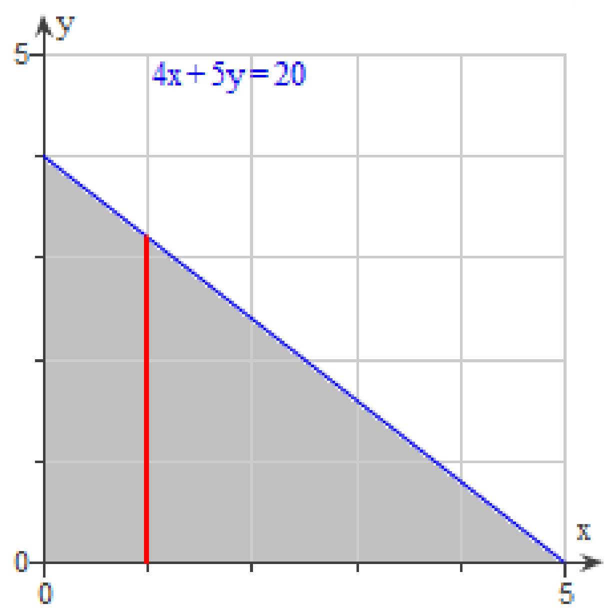
2. Find the volume of the solid generated by revolving the function about the x-axis (Note: the left side of the shape is x=0)

$$3x + 5y = 30$$

The solid is a cone with its vertex on the x-axis. A cross-section of the solid is a disk perpendicular to the x-axis with radius the segment from the x-axis to the line $4x + 5y = 20$. The volume is $\int_0^b \pi(R(x))^2 dx$ with b the x-value at which $y = 0$.

The radius as a function of x is $R(x) = \frac{20 - 4x}{5}$.

The upper limit of the volume integral is 5.



Calculus
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Visualize the shape practice

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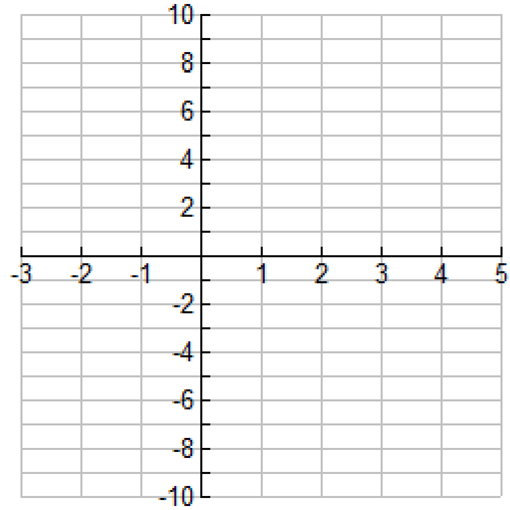
In each of problems below:

A. Sketch the shape on the graph

B. Write the formula (equation) to calculate the area described

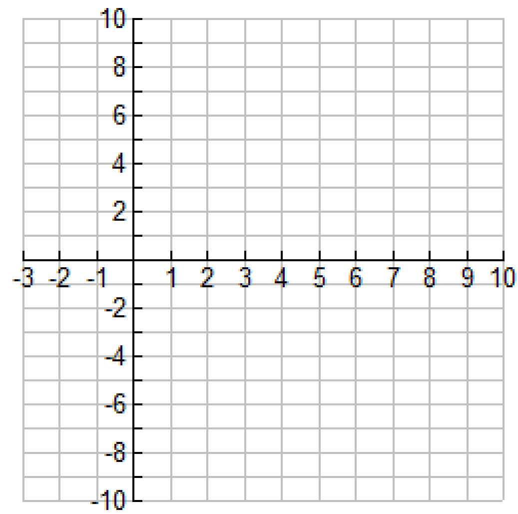
3. Find the volume of the solid generated by revolving the region bounded by $y = 5x^2$, $y = 0$, and $x = 1$ about the x-axis.

The region rotated about the x-axis to generate the solid is the shaded region in the figure.



4. Find the volume of the solid generated by revolving the region bounded by the given curve and lines about the x-axis.

$y = x, \; y = 6, \; x = 0$



Answers

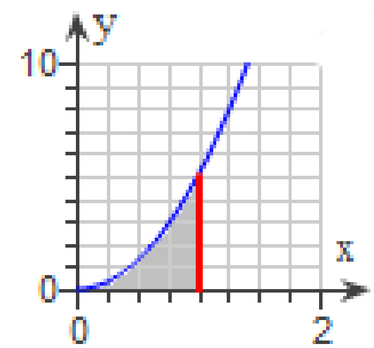
3. Find the volume of the solid generated by revolving the region bounded by $y = 5x^2$, $y = 0$, and $x = 1$ about the x-axis.

The region rotated about the x-axis to generate the solid is the shaded region in the figure.

A cross-section of the solid is a disk perpendicular to the x-axis with radius the segment from the x-axis to the curve $y = 5x^2$.

The volume, then, is $V = \int_0^1 \pi\,[R(x)]^2 dx$, where $R(x) = y$.

The volume expressed as an integral in terms of x only is $V = \int_0^1 \pi(5x^2)^2 dx$.

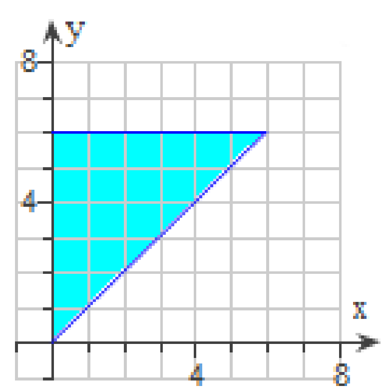


4. Find the volume of the solid generated by revolving the region bounded by the given curve and lines about the x-axis.

$y = x,\ \ y = 6,\ \ x = 0$

If the region that is revolved to generate a solid does not border on or cross the axis of revolution, the solid has a hole in it. The cross-sections perpendicular to the axis of revolution are washers instead of disks. The area of a washer is given by the following equation where R(x) is the outer radius and r(x) is the inner radius.

$$A(x) = \pi[R(x)]^2 - \pi[r(x)]^2 = \pi\left([R(x)]^2 - [r(x)]^2\right)$$



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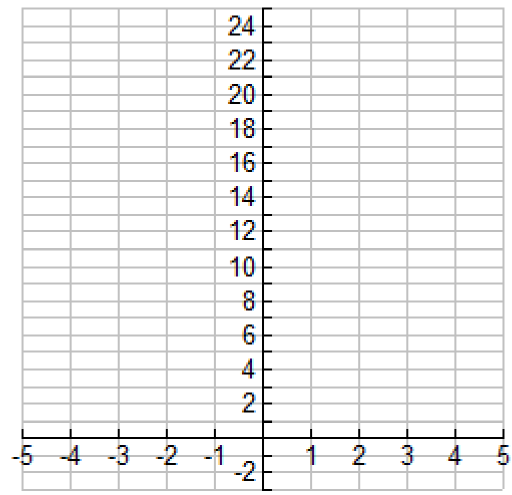
In each of problems below:

A. Sketch the shape on the graph

B. Write the formula (equation) to calculate the area described

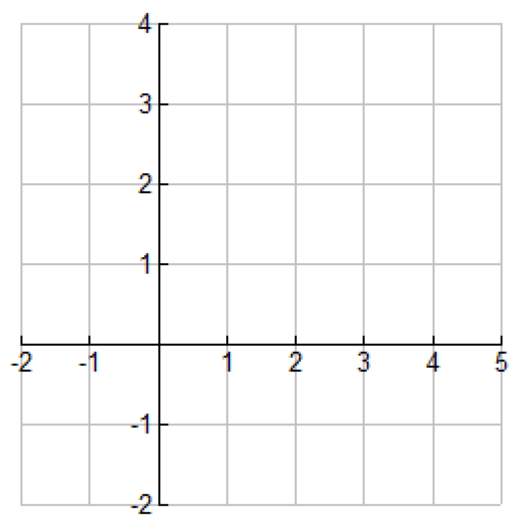
5. Find the volume of the solid generated by revolving the region bounded by the graphs of $y = 2x^2 + 1$ and $y = 2x + 8$ about the x-axis.

Begin by considering the graph of the two functions.



6. Find the volume of the solid generated by revolving the region bounded by the graphs of $y = \csc x$ and $y = 2$ about the x-axis where $0 \le x \le \pi$.

Begin by considering the graph of the two functions.



Answers:

5. Find the volume of the solid generated by revolving the region bounded by the graphs of $y = 2x^2 + 1$ and $y = 2x + 8$ about the x-axis.

Begin by considering the graph of the two functions.

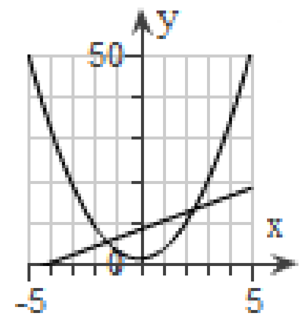
The region to be rotated about the x-axis is the region between the graphs bounded by the points of intersection of the graphs.

The graphs intersect at $x = -1.4365$ on the left and at 2.4365 on the right, rounded to four decimal places.

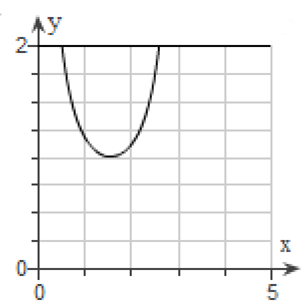
The cross-section of the solid perpendicular to the x-axis is a washer with upper radius $y_1 = 2x + 8$ and lower radius $y_2 = 2x^2 + 1$.

The area of the washer A(x), is the area of a circle with the outside radius less the area of a circle with the inside radius.

$A(x) = \pi\left[(2x+8)^2 - (2x^2+1)^2\right]$



6. Find the volume of the solid generated by revolving the region bounded by the graphs of $y = \csc x$ and $y = 2$ about the x-axis where $0 \le x \le \pi$.



Begin by considering the graph of the two functions.

The region to be rotated about the x-axis is the region between the graphs bounded by the points of intesection of the graphs.

The graph intersects at $x = \dfrac{\pi}{6}$ on the left and at $\dfrac{5\pi}{6}$ on the right.

The cross-section of the solid perpendicular to the x-axis is a washer with upper radius $y_1 = 2$ and lower radius $y_2 = \csc x$.

The area of the washer A(x), is the area of a circle with the outside radius less the area of a circle with the inside radius.

The volume, then, is $V = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \pi\left[2^2 - \csc^2 x\right]dx$.