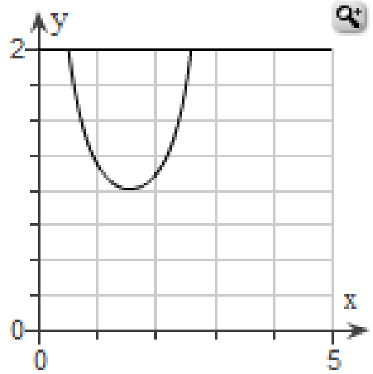
Calculus
Chap. 7 section 3
Applications of Integrals cont

Name: ____________________
Date: ____________________
Period: ____________________

1. Find the volume of the solid generated by revolving the region bounded by the graphs of $y = \csc x$ and $y = 2$ about the x-axis where $0 \le x \le \pi$.

Begin by considering the graph of the two functions.



1. answer

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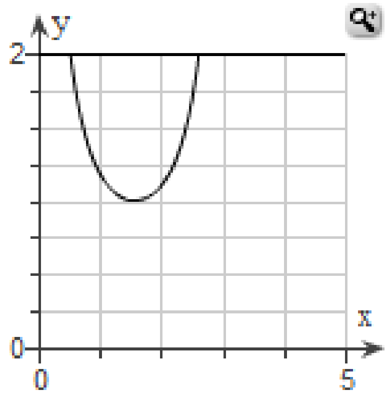

The region to be rotated about the x-axis is the region between the graphs bounded by the points of intesection of the graphs.

The graph intersects at $x = \frac{\pi}{6}$ on the left and at $\frac{5\pi}{6}$ on the right.

The cross-section of the solid perpendicular to the x-axis is a washer with upper radius $y_1 = 2$ and lower radius $y_2 = \csc x$.

The area of the washer A(x), is the area of a circle with the outside radius less the area of a circle with the inside radius.

The volume, then, is $V = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \pi\left[2^2 - \csc^2 x\right]dx$.

So, the volume is 15.44 cubic units, rounded to the nearest hundredth.

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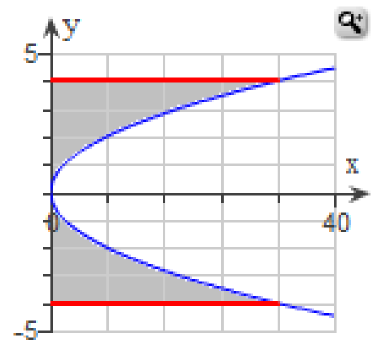
2.

Find the volume of the solid generated by revolving the region bounded by $x = 2y^2$, $x = 0$, $y = -4$, and $y = 4$ about the y-axis.

The region rotated about the y-axis to generate the solid is the shaded region in the figure.



Using the disk method the volume of revolution can be defined by the integral, $V = \int_a^b \pi[R(y)]^2 dy$, where $R(y)$ is the radius of the disk.

2. answer

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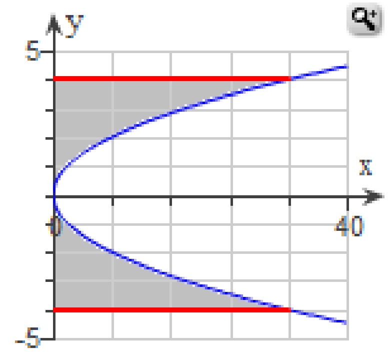

A cross-section of the solid is a disk perpendicular to the y-axis with radius the segment from the y-axis to the curve $x = 2y^2$.

The radius of the disk is $2y^2$.

The volume, then, is $V = \int_{-4}^{4} 4\pi y^4 dy$.

So, the volume is $\int_{-4}^{4} 4\pi y^4 dy = \left.\frac{4\pi y^5}{5}\right]_{-4}^{4} = \frac{8192\pi}{5}$.

Thus, $V = \frac{8192\pi}{5}$ cubic units.

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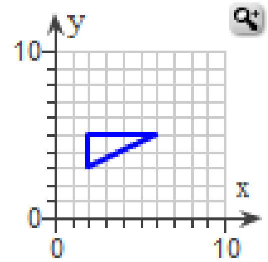
3. Find the volume of the solid generated by revolving the region enclosed by the triangle with vertices (2,3), (2,5) and (6,5) about the y-axis.

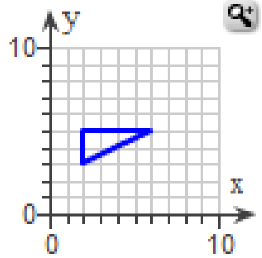
Begin by considering the graphs of segments forming the triangle.



3. answer

Find the volume of the solid generated by revolving the region enclosed by the triangle with vertices (2,3), (2,5) and (6,5) about the y-axis.

Begin by considering the graphs of segments forming the triangle.



The triangular region is to be rotated about the y-axis. Using the washer method the volume of revolution can be defined by the integral, $V = \int_3^5 \{\pi[R(y)]^2 - \pi[r(y)]^2\}\,dy$.

Cross-sections of the solid of revolution are washers perpendicular to the y-axis with constant inside radii of $r(y) = 2$.

The measures of the outside radii equal the x-coordinates of the points on the line segment joining (2,3) and (6,5). The equation of the line is $R(y) = 2y - 4$.

So, the volume is $\pi\int_3^5 [(2y-4)^2 - (2)^2]\,dy$.

Thus, the volume is $\frac{80\pi}{3}$ cubic units.

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4. Find the volume of the solid generated by revolving the following region about the y-axis.

The region in the first quadrant bounded above by the parabola $y = x^2$, below by the x-axis, and on the right by the line $x = 4$.

If the region that is revolved to generate a solid does not border on or cross the axis of revolution, the solid has a hole in it. The cross-sections perpendicular to the axis of revolution are washers instead of disks. The area of a washer is given by the following equation where R(y) is the outer radius and r(y) is the inner radius.

$$A(y) = \pi[R(y)]^2 - \pi[r(y)]^2 = \pi\left([R(y)]^2 - [r(y)]^2\right)$$

4. answer

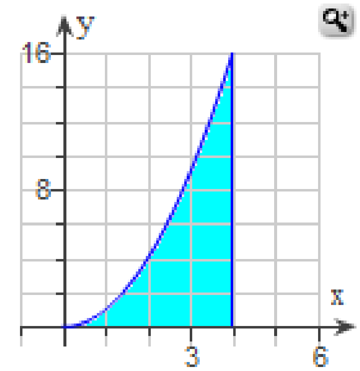
Find the volume of the solid generated by revolving the following region about the y-axis.

The region in the first quadrant bounded above by the parabola $y = x^2$, below by the x-axis, and on the right by the line $x = 4$.

If the region that is revolved to generate a solid does not border on or cross the axis of revolution, the solid has a hole in it. The cross-sections perpendicular to the axis of revolution are washers instead of disks. The area of a washer is given by the following equation where R(y) is the outer radius and r(y) is the inner radius.

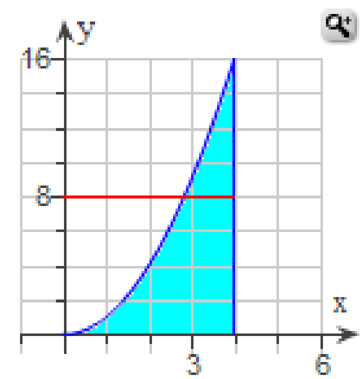
$$A(y) = \pi[R(y)]^2 - \pi[r(y)]^2 = \pi\left([R(y)]^2 - [r(y)]^2\right)$$

First sketch the planar region that is to be rotated about the y-axis. The region is shown on the right.



To find the area of the washer, determine the length of the outer and inner radii. Start by determining the length of the outer radius, R(y). The outer radius is shown on the right. The length of the outer radius is the distance from the y-axis to the line $x = 4$.

$R(y) = 4$



4. answer cont.

Substituting the lengths of the radii into the general formula for the area of a washer gives the following expression for the cross-sectional area.

$$A(y) = \pi\left([4]^2 - [\sqrt{y}\,]^2\right)$$

Simplify the expression for the cross-sectional area by squaring each term.

$$\begin{aligned} A(y) &= \pi\left([4]^2 - [\sqrt{y}\,]^2\right) \\ &= \pi(16 - y) \end{aligned}$$

The volume of a solid is given by the following formula.

$$V = \int_a^b A(y)\, dy$$

To find the volume of the solid, first determine the limits of integration. Since the planar figure is rotated about the y-axis, the integration is done with respect to y. Thus, the limits of integration are the minimum and maximum values for y.

The lower limit of integration is $a = 0$.

The upper limit of integration is $b = 16$, which is the y-coordinate of the intersection point of the parabola $y = x^2$ and the line $x = 4$.

Now integrate $y = \pi(16 - y)$ from $a = 0$ to $b = 16$ to determine the volume of the solid.

$$\begin{aligned} V &= \int_0^{16} \pi\,(16 - y)\, dy \\ &= \pi\left[16y - \frac{y^2}{2}\right]_0^{16} \\ &= 128\pi \end{aligned}$$

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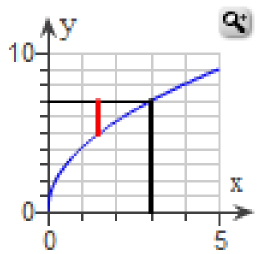
5. Find the volume of the solid generated by revolving the region bounded by $y = 4\sqrt{x}$ and the lines $y = 4\sqrt{3}$ and $x = 0$ about

a. the x-axis. **b**. the y-axis. **c**. the line $y = 4\sqrt{3}$. **d**. the line $x = 3$.

a. The red segment is the difference between the outside and inside radii of the washer used in determining the volume generated by revolving about the x-axis.



Using the washer method the volume of revolution can be defined by the integral, $V = \int_a^b \pi\left([R(x)]^2 - [r(x)]^2\right)dx$, where R(x) and r(x) are the outer and inner radius respectively.

5. answer

Find the volume of the solid generated by revolving the region bounded by $y = 4\sqrt{x}$ and the lines $y = 4\sqrt{3}$ and $x = 0$ about

a. the x-axis. **b**. the y-axis. **c**. the line $y = 4\sqrt{3}$. **d**. the line $x = 3$.

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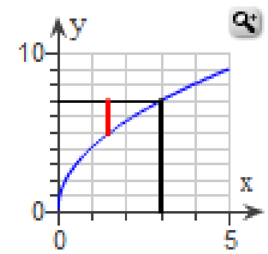
a. The red segment is the difference between the outside and inside radii of the washer used in determining the volume generated by revolving about the x-axis.



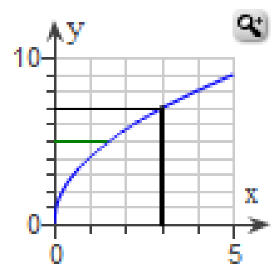
Using the washer method the volume of revolution can be defined by the integral, $V = \int_a^b \pi\left([R(x)]^2 - [r(x)]^2\right)dx$, where R(x) and r(x) are the outer and inner radius respectively.

The outside radius is $R(x) = 4\sqrt{3}$ and the inside radius is $r(x) = 4\sqrt{x}$.

So the volume is $V = \int_0^3 \pi(48 - 16x)dx$.

$$V = \int_0^3 \pi(48 - 16x)dx = \pi\,(48x - 8x^2)\big]_0^3 = 72\pi \text{ cubic units}$$

b. The green segment is the radius of the disk used in determining the volume generated by revolving about the y-axis.



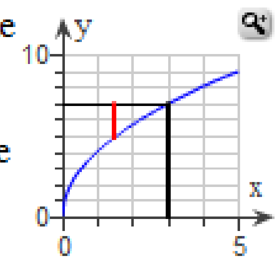
Using the disk method the volume of revolution can be defined by the integral, $V = \int_a^b \pi[R(y)]^2dy$, where R(y) is the radius of the disk.

5. answer cont.

Since the radius of the disk is $R(y) = \frac{y^2}{16}$ the volume, V, is

$$V = \int_0^{4\sqrt{3}} \pi\left(\frac{y^4}{256}\right)dy = \left[\frac{\pi y^5}{1280}\right]_0^{4\sqrt{3}} = \frac{36\pi\sqrt{3}}{5} \text{ cubic units.}$$

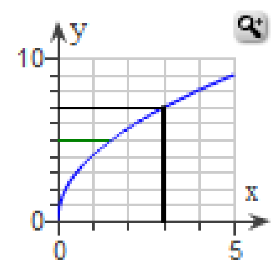
c. To determine the volume generated by rotation about $y = 4\sqrt{3}$, one should use a disk.



Using the disk method the volume of revolution can be defined by the integral, $V = \int_a^b \pi[R(x)]^2dx$, where R(x) is the radius of the disk.

Since the radius of the disk is $R(x) = 4\sqrt{3} - 4\sqrt{x}$, $V = \int_0^3 \pi\left(4\sqrt{3} - 4\sqrt{x}\right)^2dx = 24\pi$ cubic units.

d. To determine the volume generated by rotation about x = 3, one should use a washer.



Using the washer method the volume of revolution can be defined by the integral, $V = \int_a^b \pi\left([R(y)]^2 - [r(y)]^2\right)dy$, where R(y) and r(y) are the outer and inner radius respectively.

The outer radius is $R(y) = 3$. The inner radius is $r(y) = 3 - \frac{y^2}{16}$.

So, $V = \int_0^{4\sqrt{3}} \pi\left(3^2 - \left(3 - \frac{y^2}{16}\right)^2\right)dy$.

$$V = \int_0^{4\sqrt{3}} \pi\left(3^2 - \left(3 - \frac{y^2}{16}\right)^2\right)dy = \frac{84\pi\sqrt{3}}{5} \text{ cubic units}$$