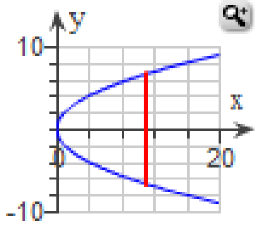
Calculus Chap. 7 section 3 Applications of Integrals	Name: ______________________ Date: ______________________ Period: ______________________

1. A solid lies between planes perpendicular to the x-axis at $x = 0$ and $x = 14$. The cross-sections perpendicular to the axis on the interval $0 \leq x \leq 14$ are squares with diagonals that run from the parabola $y = -2\sqrt{x}$ to the parabola $y = 2\sqrt{x}$. Find the volume of the solid.

First find the length of the segment, $L = f(x)$, joining opposite points on the top and bottom parabolas.



1. answer

A solid lies between planes perpendicular to the x-axis at $x = 0$ and $x = 14$. The cross-sections perpendicular to the axis on the interval $0 \le x \le 14$ are squares with diagonals that run from the parabola $y = -2\sqrt{x}$ to the parabola $y = 2\sqrt{x}$. Find the volume of the solid.

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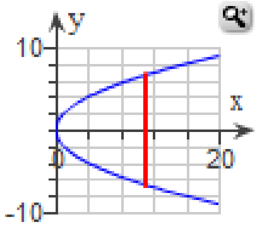


$L = f(x) = 4\sqrt{x}$

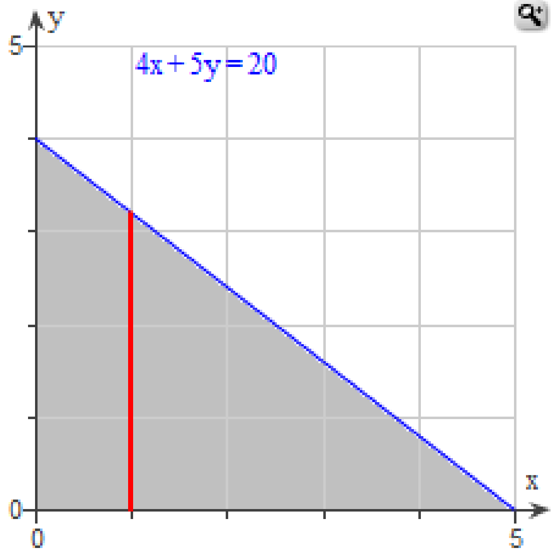
The area of the square in terms of its diagonal, D, is $\frac{D^2}{2}$.

So the area, A, of the square cross-section in terms of x is $A(x) = \frac{D^2}{2} = \frac{(4\sqrt{x})^2}{2} = 8x$.

The volume, V, of a solid of known integrable cross-sectional area A(x) from $x = a$ to $x = b$ is the integral of A from a to b. Symbolically: $V = \int_a^b A(x)dx$.

The volume in this situation is $\int_0^{14} 8x\, dx = 784$ cubic units.

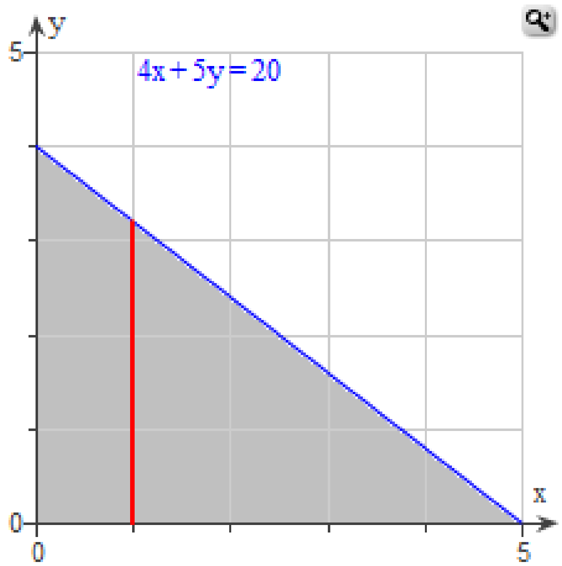
2. Find the volume of the solid generated by revolving the shaded region about the x-axis.



The solid is a cone with its vertex on the x-axis. A cross-section of the solid is a disk perpendicular to the x-axis with radius the segment from the x-axis to the line $4x + 5y = 20$. The volume is $\int_0^b \pi(R(x))^2 dx$ with b the x-value at which $y = 0$.

2. answer

Find the volume of the solid generated by revolving the shaded region about the x-axis.



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The radius as a function of x is $R(x) = \frac{20 - 4x}{5}$.

The upper limit of the volume integral is 5.

The constant multiple property of definite integrals can be used to rewrite the integral $\int_0^5 \pi\left(\frac{20-4x}{5}\right)^2 dx$ as $16\pi\int_0^5 \left(1 - \frac{x}{5}\right)^2 dx$.

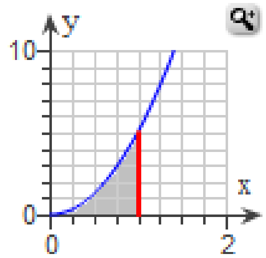
So, the volume is

$16\pi\int_0^5 \left(1 - \frac{x}{5}\right)^2 dx = \frac{80\pi}{3}$ cubic units.

3.

Find the volume of the solid generated by revolving the region bounded by $y = 5x^2$, $y = 0$, and $x = 1$ about the x-axis.

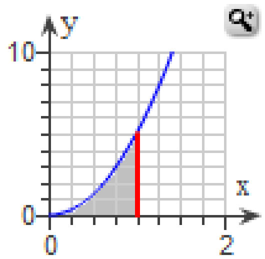
The region rotated about the x-axis to generate the solid is the shaded region in the figure.



3. answer

Find the volume of the solid generated by revolving the region bounded by $y = 5x^2$, $y = 0$, and $x = 1$ about the x-axis.

The region rotated about the x-axis to generate the solid is the shaded region in the figure.



A cross-section of the solid is a **disk** perpendicular to the x-axis with radius the segment from the x-axis to the curve $y = 5x^2$.

The volume, then, is $V = \int_0^1 \pi\,[R(x)]^2 dx$, where $R(x) = y$.

The volume expressed as an integral in terms of x only is $V = \int_0^1 \pi(5x^2)^2 dx$.

$$\int_0^1 \pi(5x^2)^2 dx = 5\pi x^5\Big]_0^1$$

Thus $V = 5\pi$ cubic units.

4. Find the volume of the solid generated by revolving the region bounded by the given curve and lines about the x-axis.

$$y = x, \;\; y = 6, \;\; x = 0$$

If the region that is revolved to generate a solid does not border on or cross the axis of revolution, the solid has a hole in it. The cross-sections perpendicular to the axis of revolution are washers instead of disks. The area of a washer is given by the following equation where R(x) is the outer radius and r(x) is the inner radius.

$$A(x) = \pi[R(x)]^2 - \pi[r(x)]^2 = \pi\left([R(x)]^2 - [r(x)]^2\right)$$

4. answer

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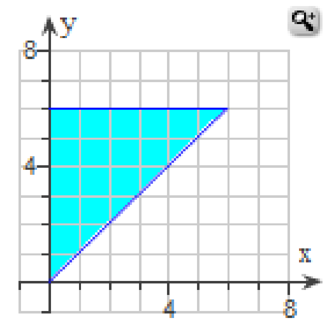
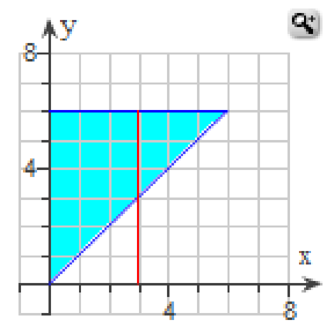
$$A(x) = \pi[R(x)]^2 - \pi[r(x)]^2 = \pi\left([R(x)]^2 - [r(x)]^2\right)$$

First sketch the planar region that is to be rotated about the x-axis. The region is shown on the right.



To find the area of the washer, determine the length of the outer and inner radii. Start by determining the length of the outer radius, R(x). The outer radius is shown on the right.

$R(x) = 6$



Substituting the lengths of the radii into the general formula for the area of a washer gives the following expression for the cross-sectional area.

$$A(x) = \pi\left([6]^2 - [x]^2\right)$$

Simplify the expression for the cross-sectional area by squaring each term.

$$\begin{aligned} A(x) &= \pi\left([6]^2 - [x]^2\right) \\ &= \pi\left(36 - x^2\right) \end{aligned}$$

The volume of a solid is given by the following formula.

$$V = \int_a^b A(x)\,dx$$

4. answer cont.

To find the volume of the solid, first determine the limits of integration. Since the planar figure is rotated about the x-axis, the integration is done with respect to x. Thus, the limits of integration are the minimum and maximum values for x.

The lower limit of integration is $a = 0$.

The upper limit of integration is $b = 6$, which is the x-coordinate of the intersection point of the two lines $y = 6$ and $y = x$.

Now integrate $A(x) = \pi\left(36 - x^2\right)$ from $a = 0$ to $b = 6$ to determine the volume of the solid.

$$V = \int_0^6 \pi\left(36 - x^2\right)\,dx$$

$$= \pi\left[36x - \frac{1}{3}x^3\right]_0^6$$

$$= 144\pi$$

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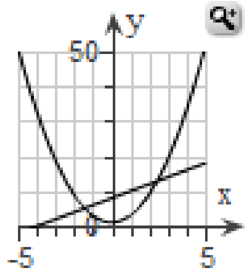
5. Find the volume of the solid generated by revolving the region bounded by the graphs of $y = 2x^2 + 1$ and $y = 2x + 8$ about the x-axis.

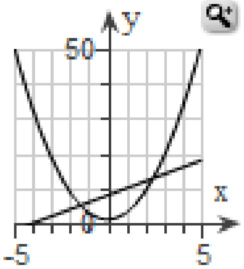
Begin by considering the graph of the two functions.

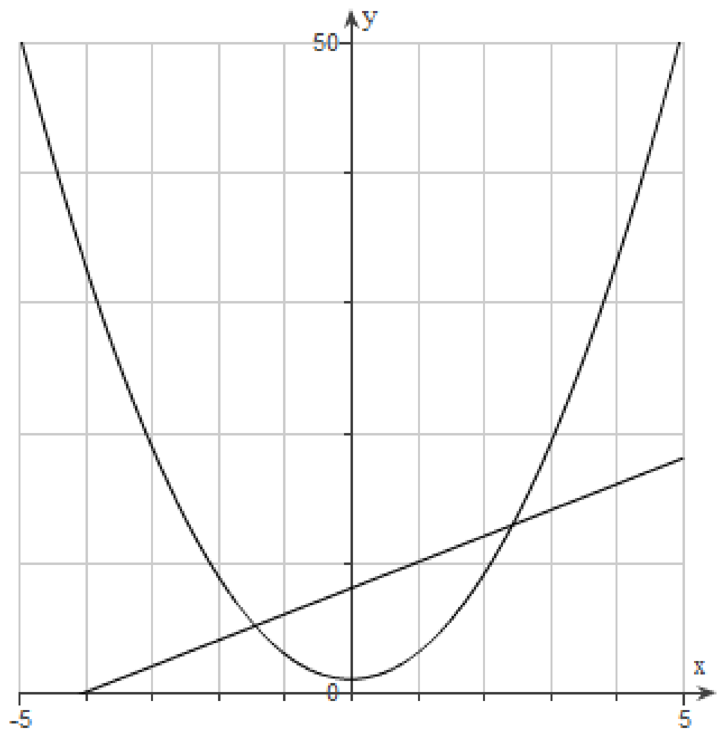


5. answer

Find the volume of the solid generated by revolving the region bounded by the graphs of $y = 2x^2 + 1$ and $y = 2x + 8$ about the x-axis.

Begin by considering the graph of the two functions.





The region to be rotated about the x-axis is the region between the graphs bounded by the points of intersection of the graphs.

The graphs intersect at $x = -1.4365$ on the left and at 2.4365 on the right, rounded to four decimal places.

The cross-section of the solid perpendicular to the x-axis is a washer with upper radius $y_1 = 2x + 8$ and lower radius $y_2 = 2x^2 + 1$.

The area of the washer A(x), is the area of a circle with the outside radius less the area of a circle with the inside radius.

$$A(x) = \pi\left[(2x+8)^2 - (2x^2+1)^2\right]$$

The volume, then, is $V = \int_{-1.4365}^{2.4365} \pi[(2x+8)^2 - (2x^2+1)^2]dx$.

So, the volume is 730.04 cubic units, rounded to the nearest hundredth.