Calculus
Chap. 7 section 2
Area between curves

Name: ______________________________
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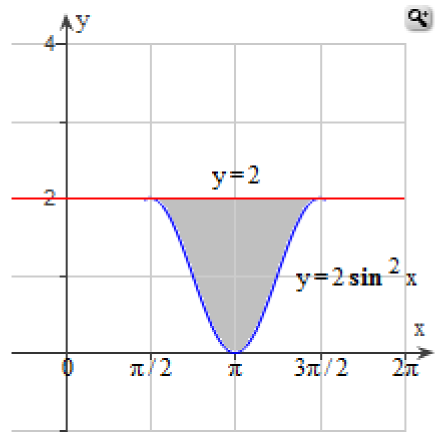
Period: ______________________________

1. If f and g are continuous with $f(x) \geq g(x)$ throughout [a,b], then the area of the region between the curves $y = f(x)$ and $y = g(x)$ from a to b is the integral of $(f - g)$ from a to b:

$$A = \int_a^b [f(x) - g(x)]dx.$$

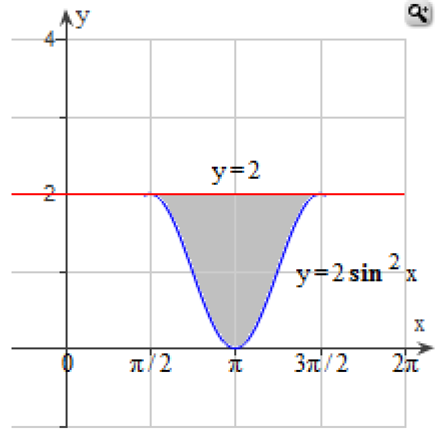
The theory is started, you continue from here….

Find the total area of the shaded region.



1. answer

Find the total area of the shaded region.



If f and g are continuous with $f(x) \geq g(x)$ throughout $[a,b]$, then the area of the region between the curves $y = f(x)$ and $y = g(x)$ from a to b is the integral of $(f - g)$ from a to b:

$$A = \int_a^b [f(x) - g(x)]dx.$$

The theory is started, you continue from here….

The upper boundary, f(x), is the curve $y = 2$.

The lower boundary, g(x), is the curve $y = 2\sin^2 x$.

The region runs from $x = \frac{\pi}{2}$ to $x = \frac{3\pi}{2}$.

Thus, the limits of integration are $a = \frac{\pi}{2}$ and $b = \frac{3\pi}{2}$.

Integrate to find the area of the shaded region.

$$A = \int_a^b [f(x) - g(x)]dx$$

$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} [(2) - (2\sin^2 x)]dx$$

$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 2(1 - \sin^2 x)dx$$

$$= 2\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos^2 x\, dx$$

Use the trigonometric identity $\cos^2 x = \frac{1 + \cos(2x)}{2}$.

$$= 2\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{1 + \cos(2x)}{2} dx$$

$$= 2\left(\frac{x}{2} + \frac{\sin(2x)}{4}\right)\Bigg]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

$$= \pi$$

Thus, the total area of the shaded region is π.

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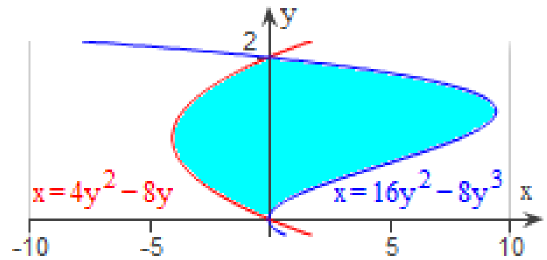
2. Find the total area of the shaded region bounded by the following curves.

$x = 16y^2 - 8y^3$
$x = 4y^2 - 8y$



If f and g are continuous with $f(y) \geq g(y)$ throughout [c, d], then the area of the region between the curves $x = f(y)$ and $x = g(y)$ from c to d is the integral of $(f - g)$ from c to d.

$$A = \int_c^d [f(y) - g(y)]dy$$

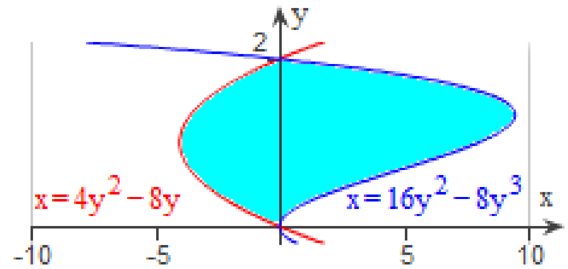
The theory is started, you continue from here….

2. answer

Find the total area of the shaded region bounded by the following curves.

$x = 16y^2 - 8y^3$

$x = 4y^2 - 8y$



If f and g are continuous with $f(y) \geq g(y)$ throughout [c, d], then the area of the region between the curves $x = f(y)$ and $x = g(y)$ from c to d is the integral of $(f - g)$ from c to d.

$$A = \int_c^d [f(y) - g(y)]dy$$

The theory is started, you continue from here….

The region's right boundary is the line $x = 16y^2 - 8y^3$, so $f(y) = 16y^2 - 8y^3$. The left boundary is the curve $x = 4y^2 - 8y$, so $g(y) = 4y^2 - 8y$.

The limits of integration can be found by looking at the graph and determining where the two functions intersect. The lower limit of integration is at $y = 0$ and the upper limit of integration is at $y = 2$.

Next set up the definite integral by substituting the functions into the formula for area.

$$A = \int_0^2 [(16y^2 - 8y^3) - (4y^2 - 8y)]dy$$

Simplifiy the definite integral.

$$A = \int_0^2 [12y^2 - 8y^3 + 8y]dy$$

Now evaluate each term. Start by integrating each term individually.

$$A = \int_0^2 12y^2\,dy - \int_0^2 8y^3\,dy + \int_0^2 8y\,dy$$
$$= [4y^3]_0^2 - [2y^4]_0^2 + [4y^2]_0^2$$

Perform the integration.

$$A = [4y^3]_0^2 - [2y^4]_0^2 + [4y^2]_0^2$$
$$= [4(2)^3 - 4(0)^3] - [2(2)^4 - 2(0)^4] + [4(2)^2 - 4(0)^2]$$
$$= 32 - 32 + 16$$
$$= 16$$

Thus, the area of the shaded region is 16.

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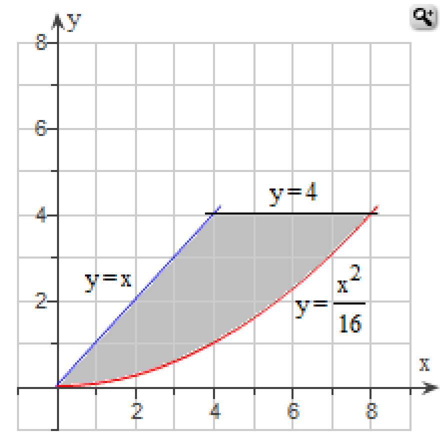
3.

If f and g are continuous with $f(x) \geq g(x)$ throughout $[a,b]$, then the area of the region between the curves $y = f(x)$ and $y = g(x)$ from a to b is the integral of $(f - g)$ from a to b:

$$A = \int_a^b [f(x) - g(x)]dx.$$

The theory is started, you continue from here….

Find the total area of the shaded region.



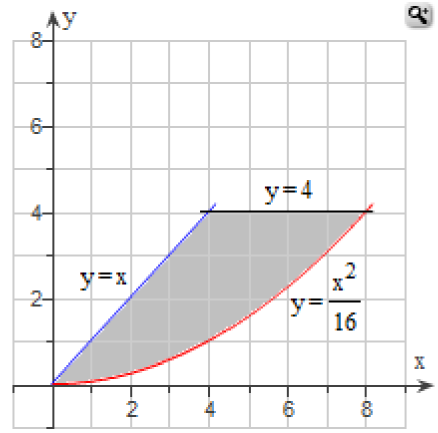
3. answer

If f and g are continuous with $f(x) \geq g(x)$ throughout $[a,b]$, then the area of the region between the curves $y = f(x)$ and $y = g(x)$ from a to b is the integral of $(f - g)$ from a to b:

$$A = \int_a^b [f(x) - g(x)]dx.$$

The theory is started, you continue from here….

Find the total area of the shaded region.



When the formula for a bounding curve changes, the area integral changes to become the sum of integrals to match.

First, find the area of the shaded region to the left of vertical line.

The upper boundary, f(x), is the curve $y = x$.

The lower boundary, g(x), is the curve $y = \frac{x^2}{16}$.

The region runs from $x = 0$ to $x = 4$.

Thus, the limits of integration are $a = 0$ and $b = 4$.

Integrate.

$$A = \int_a^b [f(x) - g(x)]dx$$

$$= \int_0^4 \left[(x) - \left(\frac{x^2}{16} \right) \right] dx$$

$$= \left(\frac{x^2}{2} - \frac{x^3}{48} \right) \Big]_0^4$$

$$= \frac{20}{3}$$

Thus, the area of the shaded region to the left of the vertical line is $\frac{20}{3}$.

Now, find the area of the shaded region to the right of the vertical line.

The upper boundary, f(x), is the curve $y = 4$. The lower boundary, g(x), is the curve $y = \frac{x^2}{16}$.

The region runs from $x = 4$ to $x = 8$. Thus, the limits of integration are $a = 4$ and $b = 8$.

Integrate.

$$A = \int_a^b [f(x) - g(x)]dx$$

$$= \int_4^8 \left[(4) - \left(\frac{x^2}{16} \right) \right] dx$$

$$= \left(4x - \frac{x^3}{48} \right) \Big]_4^8$$

$$= \frac{20}{3}$$

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4. Find the area of the region bounded by the graphs of the given equations.

$y = 4x - x^2,\ y = -12$

Start by finding the intersection points of the graphs of the functions. Solve $y = 4x - x^2$ and $y = -12$ simultaneously for x. Collect the terms on one side and factor.

$$\begin{aligned} 4x - x^2 &= -12 \\ 0 &= x^2 - 4x - 12 \\ 0 &= (x-6)(x+2) \end{aligned}$$

The theory is started, you continue from here….

4. answer

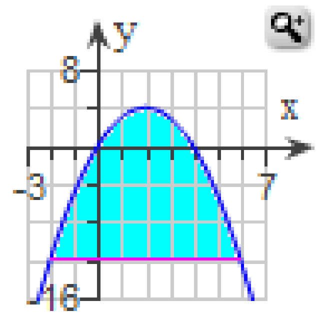
Find the area of the region bounded by the graphs of the given equations.

$y = 4x - x^2, \ y = -12$

Start by finding the intersection points of the graphs of the functions. Solve $y = 4x - x^2$ and $y = -12$ simultaneously for x. Collect the terms on one side and factor.

$$\begin{aligned} 4x - x^2 &= -12 \\ 0 &= x^2 - 4x - 12 \\ 0 &= (x-6)(x+2) \end{aligned}$$

The theory is started, you continue from here….



Now solve for x.

$$\begin{aligned} (x-6)(x+2) &= 0 \\ x &= -2,6 \end{aligned}$$

Thus, the lower limit of integration is $x = -2$ and the upper limit of integration is $x = 6$.

If f and g are continuous functions and $f(x) \geq g(x)$ over the interval [a,b], then the area of the region between the two curves from $x = a$ to $x = b$ is given by the integral below.

$$A = \int_a^b [f(x) - g(x)] \, dx$$

Next sketch the region between the curves to identify f(x) and g(x).
Identify the expressions for f(x) and g(x).

$$\begin{aligned} f(x) &= 4x - x^2 \\ g(x) &= -12 \end{aligned}$$

Next set up the definite integral by substituting the functions into the formula for area.

$$\begin{aligned} A &= \int_a^b [f(x) - g(x)] \, dx \\ &= \int_{-2}^6 \left[4x - x^2 + 12\right] dx \end{aligned}$$

Integrate the function with respect to x.

$$\int_{-2}^6 \left[4x - x^2 + 12\right] dx = \left[-\frac{1}{3}x^3 + 2x^2 + 12x\right]_{-2}^6$$

Evaluate the result at the limits of integration and subtract to find the area bounded by the curves.

$$\left[-\frac{1}{3}x^3 + 2x^2 + 12x\right]_{-2}^6 = 85\frac{1}{3}$$

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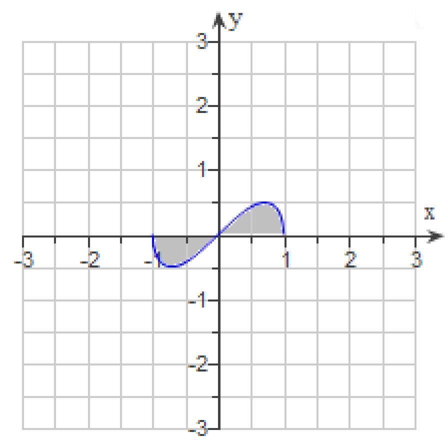
5.

Examine the graph of y. Notice this graph is symmetric about the origin. Therefore, the shaded area to the right of the y-axis has the same amount of area as the shaded area to the left of the y-axis.

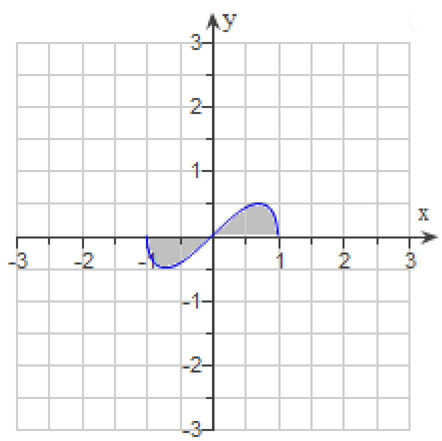
Find the total area of the shaded regions.



5. answer

Examine the graph of y. Notice this graph is symmetric about the origin. Therefore, the shaded area to the right of the y-axis has the same amount of area as the shaded area to the left of the y-axis.

Find the total area of the shaded regions.



To find the total area, find the shaded area to the right of the y-axis and multiply by 2.

That is, $A = 2\int_a^b h(x)dx$, where a is the lower limit of integration of the shaded area on the right side of the y-axis, and b is the upper limit.

The lower limit of integration of the shaded area on the right side of the y-axis is 0. The upper limit is 1.

The value of the shaded area on the right side of the y-axis is found by evaluating the following integral.

$$\int_0^1 x\sqrt{1-x^2}\,dx$$

If g′ is continuous on the interval [a,b] and f is continuous on the range of g,

then $\int_a^b f(g(x)) \cdot g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$.

Using the substitution formula, determine the function $u = g(x)$ for the integral $\int_0^1 x\sqrt{1-x^2}\,dx$.

$u = 1 - x^2$

Find du.

$du = -2x\,dx$

Solve for x dx.

$x\,dx = -\frac{1}{2}du$

Now transform the limits of integration. Recall that $u = 1 - x^2$.

When $x = 0$, $u = 1$, and when $x = 1$, $u = 0$.

Thus, the integral $\int_0^1 x\sqrt{1-x^2}\,dx$ is equivalent to the integral $\int_1^0 -\frac{1}{2}\sqrt{u}\,du$, where $u = 1 - x^2$ and $-\frac{1}{2}du = x\,dx$.

Evaluate the integral.

$$\int_1^0 -\frac{1}{2}\sqrt{u}\,du = \left[\left(-\frac{1}{2}\right)\frac{2}{3}u^{3/2}\right]_1^0 = \left[-\frac{1}{3}u^{3/2}\right]_1^0 = \frac{1}{3}$$

Now multiply the value of the shaded area on the right side of the y-axis by 2 to find the total area.

$$2 \cdot \frac{1}{3} = \frac{2}{3}$$

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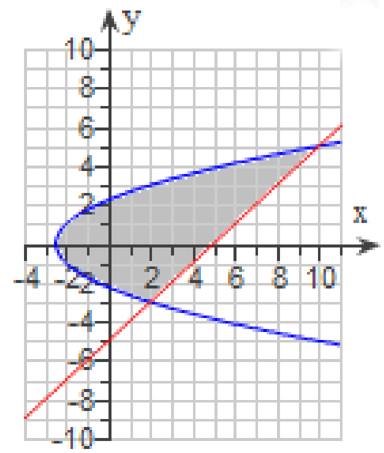
6. Find the area of the region enclosed by the curves $y^2 - 2x = 5$ and $x - y = 5$.

If f and g are continuous with $f(y) \geq g(y)$ throughout [c,d], then the area of the region between the curves $x = f(y)$ and $x = g(y)$ from c to d is the integral of $(f - g)$ from c to d:

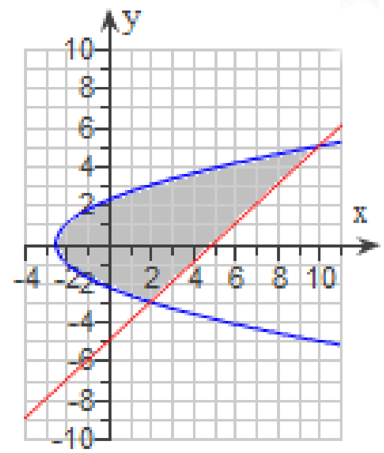
$$A = \int_c^d [f(y) - g(y)]dy.$$

A graph of the equations is shown to the right, with the enclosed region filled.



6. answer

Find the area of the region enclosed by the curves $y^2 - 2x = 5$ and $x - y = 5$.



If f and g are continuous with $f(y) \geq g(y)$ throughout [c,d], then the area of the region between the curves $x = f(y)$ and $x = g(y)$ from c to d is the integral of $(f - g)$ from c to d:

$$A = \int_c^d [f(y) - g(y)]dy.$$

A graph of the equations is shown to the right, with the enclosed region filled.

The right-hand boundary, f(y), solved for x, is the curve $x = y + 5$.

The left-hand boundary, g(y), solved for x, is the curve $x = \frac{y^2 - 5}{2}$.

The boundary points of the shaded region occur where the upper boundary and the lower boundary intersect. Find the y-values of the points of intersection for the line $x = y + 5$ and the curve $x = \frac{y^2 - 5}{2}$.

The region runs from $y = -3$ to $y = 5$. Thus, the limits of integration are $c = -3$ and $d = 5$.

Find the enclosed area.

$$A = \int_c^d [f(y) - g(y)]dy = \int_{-3}^{5} \left[(y+5) - \left(\frac{y^2 - 5}{2} \right) \right] dy$$

$$= \left(-\frac{y^3}{6} + \frac{y^2}{2} + \frac{15y}{2} \right) \Bigg]_{-3}^{5}$$

$$= \frac{128}{3}$$

Thus, the area of the region enclosed by the curves is $\frac{128}{3}$.