Calculus Chap. 7 section 1 Net Change	Name: ______________________ Date: ______________________ Period: ______________________

1. The function $v(t) = 9t^2 - 54t + 72$, $0 \le t \le 4$, is the velocity in m/sec of a particle moving along the x-axis. Use analytic methods to complete parts (a) through (c).

(a) Determine when the particle is moving to the right, to the left, and stopped.
(b) Find the particle's displacement for the given time interval. If $s(0) = 10$, what is the particle's final position?
(c) Find the total distance traveled by the particle.

(a) The particle is moving to the right when the velocity, $v(t)$, is greater than 0, to the left when the velocity is less than 0, and stopped when the velocity equals 0. Begin by setting $v(t)$ equal to 0. Then solve for t.

$$v(t) = 0$$
$$9t^2 - 54t + 72 = 0$$

1. answer

The function $v(t) = 9t^2 - 54t + 72$, $0 \le t \le 4$, is the velocity in m/sec of a particle moving along the x-axis. Use analytic methods to complete parts (a) through (c).

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(b) Find the particle's displacement for the given time interval. If $s(0) = 10$, what is the particle's final position?
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(a) The particle is moving to the right when the velocity, $v(t)$, is greater than 0, to the left when the velocity is less than 0, and stopped when the velocity equals 0. Begin by setting $v(t)$ equal to 0. Then solve for t.

$$\begin{aligned} v(t) &= 0 \\ 9t^2 - 54t + 72 &= 0 \end{aligned}$$

Factor the left side.

$$\begin{aligned} 9t^2 - 54t + 72 &= 0 \\ 9(t-2)(t-4) &= 0 \end{aligned}$$

Set each variable factor equal to 0 and solve for t.

$$\begin{aligned} t - 2 &= 0 \\ t &= 2 \end{aligned} \qquad\qquad \begin{aligned} t - 4 &= 0 \\ t &= 4 \end{aligned}$$

To determine which direction the particle is moving in each interval, test values of t in each interval. First, test a value in the interval $0 \le t < 2$. While any value in this interval can be chosen, for the purposes of this explanation, use 1. Substitute $t = 1$ in the function $v(t)$ and evaluate.

$$\begin{aligned} v(t) &= 9t^2 - 54t + 72 && \\ v(1) &= 9(1)^2 - 54(1) + 72 && \text{Substitute.} \\ &= 27 && \text{Evaluate.} \end{aligned}$$

Since $v(1) = 27$, the velocity is positive over the interval $0 \le t < 2$.

1. answer cont.

Next test a value in the interval $2 < t < 4$. While any value in this interval can be chosen, for the purposes of this explanation, use 3. Substitute $t = 3$ in the function v(t) and evaluate.

$$\begin{aligned} v(t) &= 9t^2 - 54t + 72 & \\ v(3) &= 9(3)^2 - 54(3) + 72 & \text{Substitute.} \\ &= -9 & \text{Evaluate.} \end{aligned}$$

Since $v(3) = -9$, the velocity is negative over the interval $2 < t < 4$.

Thus, the particle is moving to the right on the interval $0 \le t < 2$ seconds, to the left on the interval $2 < t < 4$ seconds, and stopped when $t = 2$ seconds and $t = 4$ seconds.

(b) The displacement is the directed distance from the particle's starting position. To find the displacement, integrate the velocity function over the given interval.

The limits of integration are the end points of the interval, 0 and 4. Find the expression to integrate.

$$\int_0^4 v(t)\,dt = \int_0^4 (9t^2 - 54t + 72)\,dt$$

Evaluate the integral.

$$\begin{aligned} \int_0^4 (9t^2 - 54t + 72)\,dt &= \left[3t^3 - 27t^2 + 72t\right]_0^4 \\ &= 48 \end{aligned}$$

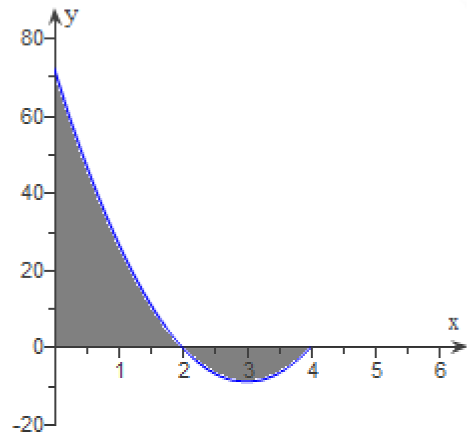
Thus, the particle's displacement for the given time interval is 48 m.

The final position of the particle is the initial position, s(0), plus the displacement. It is given in the problem statement that $s(0) = 10$. Determine the final position.

If $s(0) = 10$, the particle's final position is $10 + 48 = 58$ m.

1. answer cont.

(c) The total distance traveled does not discriminate between positive and negative direction. Record every position shift as positive by taking absolute values. To find the total distance traveled, calculate $\int_0^4 |v(t)|\,dt$.



The value of the definite integral $\int_0^4 |v(t)|\,dt = \int_0^4 |9t^2 - 54t + 72|\,dt$ can be found by adding the integral of the region above the x-axis and the absolute value of the integral of the region below the x-axis. In part **(a)** we found that $v(t) > 0$ on $[0, 2)$ and $v(t) < 0$ on $(2, 4)$. Use the expression below to compute the area.

$$\int_0^2 (9t^2 - 54t + 72)\,dt + \left| \int_2^4 (9t^2 - 54t + 72)\,dt \right|$$

Evaluate $\int_0^2 (9t^2 - 54t + 72)\,dt$.

$$\int_0^2 (9t^2 - 54t + 72)\,dt = \left[3t^3 - 27t^2 + 72t\right]_0^2$$
$$= 60$$

Next evaluate $\int_2^4 (9t^2 - 54t + 72)\,dt$.

$$\int_0^2 (9t^2 - 54t + 72)\,dt = \left[3t^3 - 27t^2 + 72t\right]_2^4$$
$$= -12$$

Compute the total area.

$$60 + |-12| = 72$$

Therefore, the total distance traveled by the particle is 72 m.

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2. The function $v(t) = \sqrt{16 - t}$, $0 \le t \le 16$ is the velocity in m/sec of a particle moving along the x-axis. Use analytic methods to do each of the following.

(a) Determine when the particle is moving to the right, to the left, and stopped.

(b) Find the particle's displacement for the given time interval. If $s(0) = 5$, what is the particle's final position?

(c) Find the total distance traveled by the particle.

(a) The direction of the motion of the particle is related to the sign of the velocity.

The particle is moving to the right when the velocity is positive.
The particle is moving to the left when the velocity is negative.
The particle is stopped when the velocity is zero.

2. answer

The function $v(t) = \sqrt{16-t}$, $0 \le t \le 16$ is the velocity in m/sec of a particle moving along the x-axis. Use analytic methods to do each of the following.
(a) Determine when the particle is moving to the right, to the left, and stopped.
(b) Find the particle's displacement for the given time interval. If $s(0) = 5$, what is the particle's final position?
(c) Find the total distance traveled by the particle.

(a) The direction of the motion of the particle is related to the sign of the velocity.

The particle is moving to the right when the velocity is positive.
The particle is moving to the left when the velocity is negative.
The particle is stopped when the velocity is zero.

The sign can change only at points where the velocity is zero. Set the expression for the velocity equal to zero and solve for t in the range $0 \le t \le 16$.

$$\begin{aligned} \sqrt{16-t} &= 0 \\ t &= 16 \end{aligned}$$

There is one point at which the velocity is zero, and it is an end point on the interval. This means that the sign over the interval is constant. The velocity $\sqrt{16-t}$ is positive in the interval $[0,16)$. Therefore, the particle is moving to the right during this interval.

The velocity is never negative in the given interval. Therefore, the particle is never moving to the left.

The velocity is zero at $t = 16$. Therefore, the particle is stopped at $t = 16$.

(b) Let Δt be a short time interval and t_k be the time of the kth interval. Assuming that the velocity is constant over this interval, the displacement will be the velocity times the time, or $v(t_k)\Delta t$.

The sum of these displacements $\sum v(t_k)\Delta t$ approximates the net displacement. As Δt goes to zero, the sum converges to the integral of the velocity over the interval.

$$\text{Displacement} = \int_{t_1}^{t_2} v(t)\, dt$$

Use an antiderivative to evaluate the integral.

2. answer cont.

$$\int_0^{16} \sqrt{16-t}\ dt = \left[-\frac{2}{3}(16-t)^{3/2} \right]_0^{16}$$

Evaluate the result at each point.

$$\left[-\frac{2}{3}(16-t)^{3/2} \right]_0^{16} = (0) - \left(-\frac{128}{3} \right)$$

Subtract to find the displacement.

$$(0) - \left(-\frac{128}{3} \right) = \frac{128}{3}\ \text{m}$$

The final position is the initial position plus the displacement. Calculate the final position.

$$5\ \text{m} + \frac{128}{3}\ \text{m} = \frac{143}{3}\ \text{m}$$

Therefore, the particle's displacement is $\frac{128}{3}$ m and its final position is $s = \frac{143}{3}$ m.

(c) The total distance can be approximated by the sum of the absolute values of the displacements, $\sum |v(t_k)|\Delta t$. This sum converges to the integral of the absolute value of the velocity as Δt approaches zero.

$$\text{Distance} = \int_{t_1}^{t_2} |v(t)|\ dt$$

Since the velocity is positive over the entire interval, then the absolute value of the velocity is equal to the velocity and the integral is the same as the integral for the displacement.

$$\int_0^{16} \left|\sqrt{16-t}\right| dt = \int_0^{16} \sqrt{16-t}\ dt = \frac{128}{3}$$

Therefore, the particle traveled a total distance of $\frac{128}{3}$ m.

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3. An automobile accelerates from rest at $8 + 3\sqrt{t}$ mph/sec for 9 seconds.

(a) What is the velocity after 9 seconds?
(b) How far does it travel in those 9 seconds.

(a) First model the effect of the acceleration on the automobile's velocity. Approximate the net change in velocity as a Riemann sum. When acceleration is constant, the velocity change equals acceleration times time.

3. answer

An automobile accelerates from rest at $8 + 3\sqrt{t}$ mph/sec for 9 seconds.

(a) What is the velocity after 9 seconds?
(b) How far does it travel in those 9 seconds.

(a) First model the effect of the acceleration on the automobile's velocity. Approximate the net change in velocity as a Riemann sum. When acceleration is constant, the velocity change equals acceleration times time.

First determine the interval on which the automobile is accelerating.

Since the automobile accelerates for 9 seconds, the interval is [0, 9].

Partition [0, 9] into short subintervals of length Δt. On each subinterval the acceleration is nearly constant, so if t_k is any point in the kth subinterval, the change in velocity imparted by the acceleration in the subinterval is approximately $a(t_k)\Delta t$ mph.

The net change in velocity for $0 \le t \le 9$ is approximately $\sum a(t_k)\Delta t$ mph.

Now use the information above to write a definite integral. The limit of these sums as the norms of the partitions go to zero is $\int_0^9 a(t)dt$.

$$\int_0^9 a(t)dt = \int_0^9 (8 + 3\sqrt{t})\, dt$$

Evaluate the integral.

$$\begin{aligned}\int_0^9 a(t)dt &= \int_0^9 (8 + 3\sqrt{t})\, dt \\ &= \int_0^9 (8 + 3t^{1/2})\, dt \\ &= \left[8t + 2t^{3/2}\right]_0^9 \\ &= 126\end{aligned}$$

Since the automobile is initially at rest, its velocity after 9 seconds is 126 mph.

(b) The Riemann sum approximating total distance traveled is $\sum |v(t_k)|\, \Delta t$, where t_k is any point in the kth subinterval.

3. answer cont.

Applying the acceleration for any length of time t adds $\int_0^t (8+3\sqrt{u})\, du$ mph to the automobile's velocity, where u is a dummy variable. Find an expression for the velocity at time t. Note that the initial velocity of the automobile is 0.

$$\begin{aligned} v(t) &= \int_0^t (8+3\sqrt{u})\, du \\ &= \int_0^t (8+3u^{1/2})\, du \\ &= 8t+2t^{3/2} \qquad \text{Evaluate.} \end{aligned}$$

Now write a definite integral for the distance traveled from $t=0$ to $t=9$ seconds.

$$\int_0^9 |v(t)|\, dt = \int_0^9 (8t+2t^{3/2})\, dt$$

Evaluate the integral.

$$\begin{aligned} \int_0^9 |v(t)|\, dt &= \int_0^9 (8t+2t^{3/2})\, dt \\ &= \left[4t^2+\frac{4}{5}t^{5/2}\right]_0^9 \\ &= 518.4 \end{aligned}$$

Convert this value from miles-per-hour second to miles by multiplying by $\frac{\text{hours}}{\text{second}} = \frac{1}{3600}$.

$$518.4 \times \frac{1}{3600} = 0.1440 \text{ mile}$$

Therefore, the automobile traveled 0.1440 mile in those 9 seconds.

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4. Recall that the acceleration due to Earth's gravity is $32\ \text{ft}/\text{sec}^2$. From ground level, a projectile is fired straight upward with velocity 100 feet per second.
(a) What is the velocity after 5 seconds?
(b) When does it hit the ground?
(c) When it hits the ground, what is the net distance it has traveled?
(d) When it hits the ground, what is the total distance it has traveled?

(a) The net velocity change of an object is the integral of the acceleration.

$$\text{Net velocity change} = \int_{t_1}^{t_2} a(t)\,dt$$

4. answer

Recall that the acceleration due to Earth's gravity is 32 ft/sec^2. From ground level, a projectile is fired straight upward with velocity 100 feet per second.
(a) What is the velocity after 5 seconds?
(b) When does it hit the ground?
(c) When it hits the ground, what is the net distance it has traveled?
(d) When it hits the ground, what is the total distance it has traveled?

(a) The net velocity change of an object is the integral of the acceleration.

$$\text{Net velocity change} = \int_{t_1}^{t_2} a(t)\ dt$$

The acceleration of the projectile is due to gravity. Since the acceleration is toward the ground, then a is $-32\ \text{ft/sec}^2$.

Write the change in velocity from 0 to 5 seconds as an integral. Use an antiderivative to evaluate the integral.

$$\int_0^5 -32\ dt = [-32t]_0^5$$

Evaluate the integral at the limits.

$$[-32t]_0^5 = (-160) - (0)$$

Subtract to find the net velocity change.

$$(-160) - (0) = -160 \text{ feet per second}$$

Add the velocity change to the initial velocity to find the velocity after 5 seconds.

$$100\ \text{ft/sec} + (-160)\ \text{ft/sec} = -60\ \text{ft/sec}$$

(b) The projectile hits the ground when s(t) becomes zero. First find an equation for the position as a function of time.

The velocity at time t is the initial velocity plus the integral of the acceleration from 0 to t. Integrate to find an equation for the velocity.

4. answer cont.

$$v(t) = 100 + \int_0^t -32\,dt$$
$$= 100 + [-32t]_0^t$$

Evaluate the integral at the limits and simplify.

$$v(t) = 100 + [-32t]_0^t$$
$$= 100 - 32t$$

The position at time t is the initial position plus the integral of the velocity from 0 to t. Integrate to find an equation for the position.

$$s(t) = \int_0^t (100 - 32t)\,dt$$
$$= \left[100t - 16t^2\right]_0^t$$

Evaluate the integral at the limits and simplify.

$$s(t) = \left[100t - 16t^2\right]_0^t$$
$$= 100t - 16t^2$$

Set the position to 0 and solve for t.

$$100t - 16t^2 = 0$$
$$t = 0, 6.25$$

Since $t = 0$ s is the time that the projectile is launched, $t = 6.25$ s is the time of impact.

(c) The net distance is the straight line distance from the initial position to the final position. Notice that the projectile travels straight up and then straight down. Therefore, the start and end positions are the same and the displacement is 0 feet.

(d) The total distance traveled is the integral of the absolute value of the velocity.

$$\text{Distance} = \int_{t_1}^{t_2} |v(t)|\,dt$$

Evaluate the integral numerically.

$$\int_0^{6.25} |100 - 32t|\,dt = 312.5$$

Therefore, the particle traveled a total distance of 312.5 m.