



Applications of Integrals

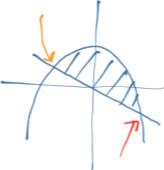
Integrals are powerful tools for solving problems.

7B Understanding Area Between Curves

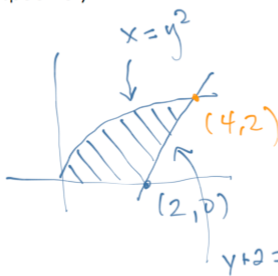
1. Set up integrals that represent the area between curves
2. Use dx and dy integrals to find areas between curves

[7.2] 3, 7, 9, 15 - 25 odd, 35, 37

Set up integrals that represent the area between curves

Sample Question	Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$
Sample Response	<p style="text-align: center;">Show / Hide Answer</p> <p>Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$</p>  <p style="text-align: center;">TOP CURVE - BOTTOM CURVE</p> <p style="text-align: center;">LIMITS</p> $-x = 2 - x^2$ $x^2 - x - 2 = 0$ $(x+1)(x-2) = 0$ $x = -1 \quad x = 2$ $\int_{-1}^2 [2 - x^2 - (-x)] dx = \left[2x - \frac{x^3}{3} + \frac{x^2}{2} \right]_{-1}^2$ $= \left[\frac{9}{2} \text{ UNITS}^2 \right]$

Use dx and dy integrals to find areas between curves

<p>Sample Question</p>	<p>Find the area of the region R in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$ by integrating with respect to y</p>
<p>Sample Response</p>	<p style="text-align: center;">Show / Hide Answer</p> <p>Find the area of the region R in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$ by integrating with respect to y</p>  <p style="margin-left: 150px;"> $x = y^2$ $x - 2 = \sqrt{x}$ $(x - 2)^2 = x$ $x^2 - 4x + 4 - x = 0$ $x^2 - 5x + 4 = 0$ $(x - 4)(x - 1)$ </p> <p style="margin-left: 150px;"> $y + 2 = x$ </p> <p style="margin-left: 100px;">RIGHT CURVE - LEFT CURVE</p> $\int_0^2 [(y+2) - y^2] dy = \left[\frac{y^2}{2} + 2y - \frac{y^3}{3} \right]_0^2$ $= \frac{10}{3} \text{ units}^2$

Find area between polar curves*

Sample Question	Find the area of the region inside enclosed by the cardioid $r = 2(1 + \cos\theta)$
Sample Response	<p style="text-align: center;">Show / Hide Answer</p> <p>Find the area of the region inside enclosed by the cardioid $r = 2(1 + \cos\theta)$</p> <p>IT TAKE 2π RADIANS TO GRAPH THE CARDIOD... (SEE CALCULATOR)</p> $\int_0^{2\pi} \frac{1}{2} r^2 d\theta = \int_0^{2\pi} \left(\frac{1}{2} \cdot 4(1 + \cos\theta)^2 \right) d\theta$ $= \int_0^{2\pi} 2(1 + 2\cos\theta + \cos^2\theta) d\theta$ $= \int_0^{2\pi} \left(2 + 4\cos\theta + 2 \frac{1 + \cos 2\theta}{2} \right) d\theta$ $= \int_0^{2\pi} (3 + 4\cos\theta + \cos 2\theta) d\theta$ $= \left[3\theta + 4\sin\theta + \frac{\sin 2\theta}{2} \right]_0^{2\pi} = \boxed{6\pi}$