



Solving Differential Equations

Challenging problems can be solved with differential equations.

6E Understanding Applications of Differential Equations

1. Solve exponential growth and decay problems
2. Solve logistic growth problems*

[6.4] 11, 15, 25, 26, 29 {6E.1}

[6.5] 5 - 11 odd, 17 {6E.2}

Solve exponential growth and decay problems

Sample Question	Suppose the amount of oil pumped from one of the canyon wells in Whittier, California, decreases at the continuous rate of 10% per year. When will the well's output fall to one-fifth of its present level?
Sample Response	<p style="text-align: center;">Show / Hide Answer</p> <p>Suppose the amount of oil pumped from one of the canyon wells in Whittier, California, decreases at the continuous rate of 10% per year. When will the well's output fall to one-fifth of its present level?</p> $\frac{dR}{dt} = k \cdot R \quad k = -0.10$ $\frac{dR}{R} = -0.10 dt$ $\int \frac{1}{R} dR = \int -0.10 dt$ $\ln R = -0.10t + C$ $ R = e^{-0.10t + C}$ $R = e^{-0.10t} \cdot e^C, \text{ let } R_0 = e^C$ $R = R_0 e^{-0.10t}$ <p>So... $\frac{1}{5} R_0 = R_0 e^{-0.10t}$</p> $\frac{1}{5} = e^{-0.10t}$ $\ln \frac{1}{5} = -0.10t$ $t = \frac{\ln \frac{1}{5}}{-0.10} \approx 16.094 \text{ yrs}$

Solve logistic growth problems*

<p>Sample Question</p>	<p>A population of rabbits is given by the formula</p> $P(t) = \frac{1000}{1 + e^{4.8 - 0.7t}}$ <p>where t is the number of months after a few rabbits are released.</p> <p>- Show that the function is the solution of a logistic differential equation and identify the carrying capacity</p>
<p>Sample Response</p>	<p style="text-align: center;">Show / Hide Answer</p> <p>A population of rabbits is given by the formula</p> $P(t) = \frac{1000}{1 + e^{4.8 - 0.7t}}$ <p>where t is the number of months after a few rabbits are released.</p> <p>- Show that the function is the solution of a logistic differential equation and identify the carrying capacity</p> $\frac{dP}{dt} = \frac{k}{M} P(M - P), \quad P = \frac{M}{1 + Ae^{-kt}}$ <p>ARE THE TWO FORMS OF A LOGISTIC FUNCTION...</p> $1 + e^{4.8 - 0.7t} = 1 + Ae^{-kt}$ $1 + \boxed{e^{4.8}} \cdot e^{-0.7t}$ <p style="text-align: center;"> ↑ ↑ "A" "k" </p>