Calculus
Chap. 6 section 4
Targets with examples

Name: _____
Date: _____
Period: _____

Solving Differential Equations

Challenging problems can be solved with differential equations.

6C Understanding Techniques for Integration

1. Integrate by substitution
2. Integrate by parts*
3. Use partial fraction decomposition to integrate*

[6.2] 1 - 37 odd {6C.1}
[6.3] 1 - 21 odd {6C.2}
[8.4] 19 - 29 odd {6C.3}

Integrate by substitution

Sample Question	Evaluate the following integral using substitution $\int(\sqrt{4x-1})dx$
Sample Response	Show / Hide Answer Evaluate the following integral using substitution $\int(\sqrt{4x-1})dx$ let $u = 4x-1$ $\frac{du}{dx} = 4, du = 4dx$ $\frac{1}{4}\int\sqrt{u}\,du$ $= \frac{1}{4}\cdot\frac{2}{3}u^{3/2} + C$ $= \frac{1}{6}u^{3/2} + C = \boxed{\frac{1}{6}(4x-1)^{3/2} + C}$

Integrate by parts*

Sample Question	Use integration by parts to evaluate the following integral $\int (x^2 \sin x)dx$
Sample Response	Show / Hide Answer Use integration by parts to evaluate the following integral $\int (x^2 \sin x)dx$ $u = x^2 \quad dv = \sin x\,dx$ $du = 2x\,dx \quad v = -\cos x$ $\int u\,dv = uv - \int v\,du$ $\int x^2 \sin = -x^2 \cos x + \int \cos x \cdot 2x\,dx$ $= -x^2 \cos x + [\quad]$
	$u = 2x \quad dv = \cos x\,dx$ $du = 2dx \quad v = \sin x$ $= x^2 \cos x + [2x \sin x - \int 2 \sin x\,dx]$ $= x^2 \cos x + 2x \sin x + 2 \cos x$

Use partial fraction decomposition to integrate*

Sample Question	Use integration by partial fraction decomposition to evaluate the following integral $\int\left(\frac{1}{x^2-4}\right)dx$
Sample Response	Show / Hide Answer *SOLUTION 1 :* Integrate $\int \frac{1}{x^2-4}\,dx$. Factor and decompose into partial fractions, getting $\int \frac{1}{x^2-4}\,dx = \int \frac{1}{(x+2)(x-2)}\,dx$ $= \int \left(\frac{A}{x+2}+\frac{B}{x-2}\right)dx$ (After getting a common denominator, adding fractions, and equating numerators, it follows that $A(x-2)+B(x+2)=1$; let $x=-2:\ A(-4)+B(0)=1 \longrightarrow A=-\frac{1}{4}$; let $x=2:\ A(0)+B(4)=1 \longrightarrow B=\frac{1}{4}$.) $= \int \left(\frac{-1/4}{x+2}+\frac{1/4}{x-2}\right)dx$ $= \int \left(-(1/4)\frac{1}{x+2}+(1/4)\frac{1}{x-2}\right)dx$ $= -\frac{1}{4}\ln\lvert x+2\rvert+\frac{1}{4}\ln\lvert x-2\rvert+C$ $= \frac{1}{4}\left(\ln\lvert x-2\rvert-\ln\lvert x+2\rvert\right)+C$ (Recall that $\ln m-\ln n=\ln\left(\frac{m}{n}\right)$.) $= \frac{1}{4}\ln\frac{\lvert x-2\rvert}{\lvert x+2\rvert}+C$. From www.math.ucdavis.edu