Calculus
Chap. 6 section 6
Logistics

Name: ______________________________
Date: ______________________________
Period: ______________________________

1. Solve the initial value problem using partial fractions. Use a graphing utility to generate a slope field for the differential equation and verify that the solution conforms to the slope field.

$\frac{dP}{dt} = .007P(700 - P)$ and $P = 9$ when $t = 0$.

To solve the differential equation, begin by separating the variables in the differential equation. Treat $\frac{dP}{dt}$ as a quotient of differentials, and multiply both sides by dt.

$dP = .007P(700 - P)dt$

The theory is started, you continue from here….

1. answer

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Next divide both sides of the equation by $P(700 - P)$. This will separate the variables in the differential equation and make it possible to integrate.

$$\begin{aligned} dP &= .007P(700 - P)dt \\ \frac{dP}{P(700 - P)} &= .007dt \end{aligned}$$

Integrate each side of the equation, beginning with the left side. First use partial fraction decomposition to rewrite the fraction in a form that can be integrated.

$$\begin{aligned} \int \frac{1}{P(700 - P)} dP &= \int .007dt \\ \int \left(\frac{A}{P} + \frac{B}{700 - P} \right) dP &= \int .007dt \end{aligned}$$

From above, it must be true that $A(700 - P) + B(P) = 1$. Use this fact in order to find A and B. To find A, let $P = 0$ and solve for A.

$$\begin{aligned} A(700 - (0)) + B(0) &= 1 \\ A &= \frac{1}{700} \end{aligned}$$

Now to find B, let $P = 700$ and solve for B.

$$\begin{aligned} A(700 - (700)) + B(700) &= 1 \\ B &= \frac{1}{700} \end{aligned}$$

1. answer cont.

With these values of A and B, rewrite the integral on the left side.

$$\int\left(\frac{\left(\frac{1}{700}\right)}{P}+\frac{\left(\frac{1}{700}\right)}{700-P}\right)dP = \int .007dt$$

$$\frac{1}{700}\int\left(\frac{1}{P}+\frac{1}{700-P}\right)dP = .007\int dt$$

Move the constants to the outside.

Multiply both sides of the equation by 700.

$$\int\left(\frac{1}{P}+\frac{1}{700-P}\right)dP = 4.9\int dt$$

Now integrate on the left side.

$$\int\left(\frac{1}{P}+\frac{1}{700-P}\right)dP = 4.9\int dt$$

$$\mathbf{ln}\,|P| - \mathbf{ln}\,|700-P| + C_1 = 4.9\int dt$$

Next integrate on the right side.

$$\mathbf{ln}\,|P| - \mathbf{ln}\,|700-P| + C_1 = 4.9\int dt$$

$$\mathbf{ln}\,|P| - \mathbf{ln}\,|700-P| + C_1 = 4.9t + C_2$$

To save extra calculations later on, divide each side of the equation by -1 now. Then write the expression on the left as a single logarithm.

$$\mathbf{ln}\,|700-P| - \mathbf{ln}\,|P| = -4.9t - C$$

$$\mathbf{ln}\left|\frac{700-P}{P}\right| = -4.9t - C$$

Combine the constants into one arbitrary constant, C.

Next exponentiate both sides.

$$e^{\mathbf{ln}\left|\frac{700-P}{P}\right|} = e^{-4.9t-C}$$

$$\frac{700}{P} - 1 = e^{-4.9t}e^{-C}$$

$$e^{\mathbf{ln}\,a} = a$$

$$\frac{700}{P} = 1 + e^{-4.9t}e^{-C}$$

1. answer cont.

Use the initial conditions to find e^{-C}. Substitute 9 for P and 0 for t and solve for e^{-C}.

$$\frac{700}{9} = 1 + e^{-4.9(0)} e^{-C}$$
$$76.778 \approx e^{-C}$$

Substitute this value for e^{-C} into the equation and solve for P.

$$\frac{700}{P} \approx 1 + (76.778)\, e^{-4.9t}$$
$$P \approx \frac{700}{1 + 76.778\, e^{-4.9t}}$$

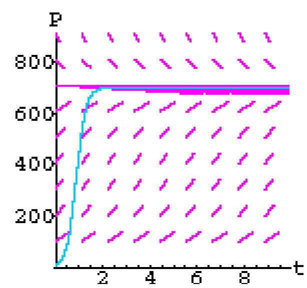
Thus, the solution to the differential equation is $P \approx \dfrac{700}{1 + 76.778\, e^{-4.9t}}$.

Next determine the slope field for the differential equation. The differential equation $\dfrac{dP}{dt} = .007P(700 - P)$ gives the slope at different values of P.

Substitute several values of P into the differential equation $\dfrac{dP}{dt} = .007P(700 - P)$ to obtain an idea of what the slope fields looks like. First notice that when $P = 0$ or $P = 700$, the slope is 0.

Also notice that when P is between 0 and 700, the slope is positive and when P is greater than 700, the slope is negative.

The slope field of the differential equation is shown on the right. Notice that the solution to the differential equation (blue curve) conforms nicely to the slope field.



Next integrate on the right side.

$$\ln|P| - \ln|700-P| + C_1 = 4.9\int dt$$
$$\ln|P| - \ln|700-P| + C_1 = 4.9t + C_2$$

To save extra calculations later on, divide each side of the equation by -1 now. Then write the expression on the left as a single logarithm.

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