Calculus
Chap. 6 section 6
Eulers method example worked out!

Name: ________________________________
Date: ________________________________
Period: ________________________________

1. Use Euler's Method with increments of $\Delta x = 0.1$ to approximate the value of y when $x = 1.3$.

$\frac{dy}{dx} = x - 1$ and $y = 6$ when $x = 1$

Use Euler's Method to construct an approximation of the curve from $x = 1$ to $x = 1.3$, pasting together three small linearization segments. Each segment will extend from a point (x,y) to a point $(x + \Delta x, y + \Delta y)$, where $\Delta x = 0.1$ and $\Delta y = \left(\frac{dy}{dx}\right)\Delta x$.

1. answer

Use Euler's Method with increments of $\Delta x = 0.1$ to approximate the value of y when $x = 1.3$.

$$\frac{dy}{dx} = x - 1 \text{ and } y = 6 \text{ when } x = 1$$

Use Euler's Method to construct an approximation of the curve from $x = 1$ to $x = 1.3$, pasting together three small linearization segments. Each segment will extend from a point (x,y) to a point $(x + \Delta x, y + \Delta y)$, where $\Delta x = 0.1$ and $\Delta y = \left(\frac{dy}{dx}\right)\Delta x$.

The table below shows how to construct each new point from the previous one. Δx is the same for each segment. Begin by calculating $\frac{dy}{dx}$ for the initial point.

(x,y)	$\frac{dy}{dx} = x - 1$	Δx	$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$	$(x + \Delta x, y + \Delta y)$
$(1,6)$	$1 - 1 = 0$	0.1		

Now calculate Δy using the values for $\frac{dy}{dx}$ and Δx.

(x,y)	$\frac{dy}{dx} = x - 1$	Δx	$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$	$(x + \Delta x, y + \Delta y)$
$(1,6)$	0	0.1	$0 \cdot 0.1 = 0$	

To determine the next point, find $(x + \Delta x, y + \Delta y)$.

(x,y)	$\frac{dy}{dx} = x - 1$	Δx	$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$	$(x + \Delta x, y + \Delta y)$
$(1,6)$	0	0.1	0	$(1 + 0.1, 6 + 0) = (1.1, 6)$

Use this point to calculate the values for the second segment.

(x,y)	$\frac{dy}{dx} = x - 1$	Δx	$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$	$(x + \Delta x, y + \Delta y)$
$(1,6)$	0	0.1	0	$(1.1, 6)$
$(1.1, 6)$	$1.1 - 1 = 0.1$	0.1	$0.1 \cdot 0.1 = 0.01$	$(1.1 + 0.1, 6 + 0.01) = (1.2, 6.01)$

Now use the point $(1.2, 6.01)$ to find the third and final segment.

(x,y)	$\frac{dy}{dx} = x - 1$	Δx	$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$	$(x + \Delta x, y + \Delta y)$
$(1,6)$	0	0.1	0	$(1.1, 6)$
$(1.1, 6)$	0.1	0.1	0.01	$(1.2, 6.01)$
$(1.2, 6.01)$	$1.2 - 1 = 0.2$	0.1	$0.2 \cdot 0.1 = 0.02$	$(1.2 + 0.1, 6.01 + 0.02) = (1.3, 6.03)$

Since the value of $x = 1.3$ has been reached, stop any further calculations. Thus, Euler's Method leads to the approximation $y \approx 6.03$.

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2. Use Euler's Method with increments of $\Delta x = 0.1$ to approximate the value of y when $x = 1.3$.

$$\frac{dy}{dx} = y - x \text{ and } y = 6 \text{ when } x = 1$$

Use Euler's Method to construct an approximation of the curve from $x = 1$ to $x = 1.3$, pasting together three small linearization segments. Each segment will extend from a point (x,y) to a point $(x + \Delta x, y + \Delta y)$, where $\Delta x = 0.1$ and $\Delta y = \left(\frac{dy}{dx}\right)\Delta x$.

2. answer

Use Euler's Method with increments of $\Delta x = 0.1$ to approximate the value of y when $x = 1.3$.

$$\frac{dy}{dx} = y - x \text{ and } y = 6 \text{ when } x = 1$$

Use Euler's Method to construct an approximation of the curve from $x = 1$ to $x = 1.3$, pasting together three small linearization segments. Each segment will extend from a point (x,y) to a point $(x + \Delta x, y + \Delta y)$, where $\Delta x = 0.1$ and $\Delta y = \left(\frac{dy}{dx}\right)\Delta x$.

The table below shows how to construct each new point from the previous one. Note that Δx is the same for each segment. Begin by calculating $\frac{dy}{dx}$ for the initial point.

(x,y)	$\frac{dy}{dx} = y - x$	Δx	$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$	$(x + \Delta x, y + \Delta y)$
(1,6)	$6 - 1 = 5$	0.1		

Now calculate Δy using the values for $\frac{dy}{dx}$ and Δx given in the first row.

(x,y)	$\frac{dy}{dx} = y - x$	Δx	$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$	$(x + \Delta x, y + \Delta y)$
(1,6)	5	0.1	$5 \cdot 0.1 = 0.5$	

To determine the next point, find $(x + \Delta x, y + \Delta y)$.

(x,y)	$\frac{dy}{dx} = y - x$	Δx	$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$	$(x + \Delta x, y + \Delta y)$
(1,6)	5	0.1	0.5	$(1 + 0.1, 6 + 0.5) = (1.1, 6.5)$

Use the point (1.1,6.5) to calculate the values in the second row.

(x,y)	$\frac{dy}{dx} = y - x$	Δx	$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$	$(x + \Delta x, y + \Delta y)$
(1,6)	5	0.1	0.5	(1.1,6.5)
(1.1,6.5)	$6.5 - 1.1 = 5.4$	0.1	$5.4 \cdot 0.1 = 0.54$	$(1.1 + 0.1, 6.5 + 0.54) = (1.2, 7.04)$

2. answer cont.

Now use the point (1.2,7.04) to find the third and final segment.

(x,y)	$\frac{dy}{dx} = y - x$	Δx	$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$	$(x + \Delta x, y + \Delta y)$
(1,6)	5	0.1	0.5	(1.1,6.5)
(1.1,6.5)	5.4	0.1	0.54	(1.2,7.04)
(1.2,7.04)	$7.04 - 1.2 = 5.84$	0.1	$5.84 \cdot 0.1 = 0.584$	$(1.2 + 0.1, 7.04 + 0.584) = (1.3, 7.624)$

Since the value of $x = 1.3$ has been reached, stop any further calculations. Thus, Euler's Method leads to the approximation $y \approx 7.624$. That is, the value at $x = 1.3$ is approximately 7.624.

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3. Use Euler's Method with increments of $\Delta x = -0.1$ to approximate the value of y when $x = 1.7$.

$$\frac{dy}{dx} = x - y \text{ and } y = 6 \text{ when } x = 2$$

Use Euler's Method to construct an approximation of the curve from $x = 2$ to $x = 1.7$, pasting together three small linearization segments. Each segment will extend from a point (x,y) to a point $(x + \Delta x, y + \Delta y)$, where $\Delta x = -0.1$ and $\Delta y = \left(\frac{dy}{dx}\right)\Delta x$.

3. answer

Use Euler's Method with increments of $\Delta x = -0.1$ to approximate the value of y when $x = 1.7$.

$$\frac{dy}{dx} = x - y \text{ and } y = 6 \text{ when } x = 2$$

Use Euler's Method to construct an approximation of the curve from $x = 2$ to $x = 1.7$, pasting together three small linearization segments. Each segment will extend from a point (x,y) to a point $(x + \Delta x, y + \Delta y)$, where $\Delta x = -0.1$ and $\Delta y = \left(\frac{dy}{dx}\right)\Delta x$.

The table below shows how to construct each new point from the previous one. Note that Δx is the same for each segment. Begin by calculating $\frac{dy}{dx}$ for the initial point.

(x,y)	$\frac{dy}{dx} = x - y$	Δx	$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$	$(x + \Delta x, y + \Delta y)$
$(2,6)$	$2 - 6 = -4$	-0.1		

Now calculate Δy using the values for $\frac{dy}{dx}$ and Δx given in the first row.

(x,y)	$\frac{dy}{dx} = x - y$	Δx	$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$	$(x + \Delta x, y + \Delta y)$
$(2,6)$	-4	-0.1	$-4 \cdot -0.1 = 0.4$	

To determine the next point, find $(x + \Delta x, y + \Delta y)$.

(x,y)	$\frac{dy}{dx} = x - y$	Δx	$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$	$(x + \Delta x, y + \Delta y)$
$(2,6)$	-4	-0.1	0.4	$(2 - 0.1, 6 + 0.4) = (1.9, 6.4)$

Use the point $(1.9,6.4)$ to calculate the values in the second row.

(x,y)	$\frac{dy}{dx} = x - y$	Δx	$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$	$(x + \Delta x, y + \Delta y)$
$(2,6)$	-4	-0.1	0.4	$(1.9,6.4)$
$(1.9,6.4)$	$1.9 - 6.4 = -4.5$	-0.1	$-4.5 \cdot -0.1 = 0.45$	$(1.9 - 0.1, 6.4 + 0.45) = (1.8, 6.85)$

3. answer cont.

Now use the point (1.8,6.85) to find the third and final segment.

(x,y)	$\frac{dy}{dx} = x - y$	Δx	$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$	$(x + \Delta x, y + \Delta y)$
(2,6)	-4	-0.1	0.4	(1.9,6.4)
(1.9,6.4)	-4.5	-0.1	0.45	(1.8,6.85)
(1.8,6.85)	$1.8 - 6.85 = -5.05$	-0.1	$-5.05 \cdot -0.1 = 0.505$	$(1.8 - 0.1, 6.85 + 0.505) =$ (1.7,7.355)

Since the value of $x = 1.7$ has been reached, stop any further calculations. Thus, Euler's Method leads to the approximation $y \approx 7.355$. That is, the value at $x = 1.7$ is approximately 7.355.

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4. Let $y = f(x)$ be the solution to the initial value problem $dy/dx = 2x + 1$ such that $f(7) = 3$. Find the percentage error if Euler's Method with $\Delta x = 0.1$ is used to approximate $f(7.4)$.

If the general solution to a first-order differential equation is continuous, the only additional information needed is the value of the function at a single point, called an initial condition. A differential equation with an initial condition is called an initial value problem. It has a unique solution, called the particular solution to the differential equation.

4. answer

Let $y = f(x)$ be the solution to the initial value problem $dy/dx = 2x + 1$ such that $f(7) = 3$. Find the percentage error if Euler's Method with $\Delta x = 0.1$ is used to approximate $f(7.4)$.

If the general solution to a first-order differential equation is continuous, the only additional information needed is the value of the function at a single point, called an initial condition. A differential equation with an initial condition is called an initial value problem. It has a unique solution, called the particular solution to the differential equation.

To find the solution to the initial value problem, first find the general solution for y.

$$y = x^2 + x + C$$

Next, apply the initial condition, $f(7) = 3$, to find C.

$$\begin{aligned} y &= x^2 + x + C \\ 3 &= 7^2 + 7 + C \\ -53 &= C \end{aligned}$$

Thus, the particular solution is $y = x^2 + x - 53$, or $f(x) = x^2 + x - 53$.

Use the particular solution to find $f(7.4)$.

$$\begin{aligned} f(x) &= x^2 + x - 53 \\ f(7.4) &= 7.4^2 + 7.4 - 53 \\ f(7.4) &= 9.16 \end{aligned}$$

Next, use Euler's Method to construct an approximation of the curve from $x = 7$ to $x = 7.4$, pasting together four small linearization segments. Each segment will extend from a point (x,y) to a point $(x + \Delta x, y + \Delta y)$, where $\Delta x = 0.1$ and $\Delta y = \left(\frac{dy}{dx}\right)\Delta x$.

The table below shows how to construct each new point from the previous one. Note that Δx is the same for each segment. Begin by calculating $\frac{dy}{dx}$ for the initial point.

(x,y)	$\frac{dy}{dx} = 2x + 1$	Δx	$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$	$(x + \Delta x, y + \Delta y)$
$(7,3)$	$2(7) + 1 = 15$	0.1		

Now calculate Δy using the values for $\frac{dy}{dx}$ and Δx given in the first row.

(x,y)	$\frac{dy}{dx} = 2x + 1$	Δx	$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$	$(x + \Delta x, y + \Delta y)$
$(7,3)$	15	0.1	$15 \cdot 0.1 = 1.5$	

4. answer cont.

To determine the next point, find $(x + \Delta x, y + \Delta y)$.

(x,y)	$\frac{dy}{dx} = 2x + 1$	Δx	$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$	$(x + \Delta x, y + \Delta y)$
(7,3)	15	0.1	1.5	$(7 + 0.1, 3 + 1.5) =$ (7.1,4.5)

Use (7.1,4.5) to calculate the values in the second row. Find $(x + \Delta x, y + \Delta y)$.

(x,y)	$\frac{dy}{dx} = 2x + 1$	Δx	$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$	$(x + \Delta x, y + \Delta y)$
(7,3)	15	0.1	1.5	(7.1,4.5)
(7.1,4.5)	$2(7.1) + 1 = 15.2$	0.1	$15.2 \cdot 0.1 = 1.52$	$(7.1 + 0.1, 4.5 + 1.52) =$ (7.2,6.02)

Use (7.2,6.02) to calculate the values in the third row. Calculate $(x + \Delta x, y + \Delta y)$.

(x,y)	$\frac{dy}{dx} = 2x + 1$	Δx	$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$	$(x + \Delta x, y + \Delta y)$
(7,3)	15	0.1	1.5	(7.1,4.5)
(7.1,4.5)	15.2	0.1	1.52	(7.2,6.02)
(7.2,6.02)	$2(7.2) + 1 = 15.4$	0.1	$15.4 \cdot 0.1 = 1.54$	$(7.2 + 0.1, 7.56 + 1.54) =$ (7.3,7.56)

Finally, use (7.3,7.56) to calculate the values in the fourth and final row. Find $(x + \Delta x, y + \Delta y)$.

(x,y)	$\frac{dy}{dx} = 2x + 1$	Δx	$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$	$(x + \Delta x, y + \Delta y)$
(7,3)	15	0.1	1.5	(7.1,4.5)
(7.1,4.5)	15.2	0.1	1.52	(7.2,6.02)
(7.2,6.02)	15.4	0.1	1.54	(7.3,7.56)
(7.3,7.56)	$2(7.3) + 1 = 15.6$	0.1	$15.6 \cdot 0.1 = 1.56$	$(7.3 + 0.1, 7.56 + 1.56) =$ (7.4,9.12)

Since the value of $x = 7.4$ has been reached, stop any further calculations. Thus, Euler's Method leads to the approximation $f(7.4) \approx 9.12$.

Now, find the percentage error given that $f(7.4) = 9.16$ and Euler's Method leads to the approximation $f(7.4) \approx 9.12$, rounding to one decimal place.

$$\frac{9.16 - 9.12}{9.16} \cdot 100\% = 0.4\%$$

Thus, the percentage error if Euler's Method with $\Delta x = 0.1$ is used to approximate $f(7.4)$ is 0.4%.