Calculus Chap. 6 section 4 Integration by parts	Name: ________________ Date: ________________ Period: ________________

1. Evaluate $\int e^{2x} \cos(3x)\,dx$.

To evaluate $\int e^{2x} \cos(3x)\,dx$, use integration by parts repeatedly until the original integral recurs. The resulting equation can then be solved for the original integral.

1. answer

Evaluate $\int e^{2x}\cos(3x)dx$.

To evaluate $\int e^{2x}\cos(3x)dx$, use integration by parts repeatedly until the original integral recurs. The resulting equation can then be solved for the original integral.

Begin with $u = \cos(3x)$.

$$u = \cos(3x) \qquad du = -3\sin(3x)dx$$
$$v = \frac{1}{2}e^{2x} \qquad dv = e^{2x}dx$$

Use integration by parts.

$$\int e^{2x}\cos(3x)dx = \frac{1}{2}e^{2x}\cos(3x) - \int -\frac{3}{2}e^{2x}\sin(3x)dx$$
$$= \frac{1}{2}e^{2x}\cos(3x) + \int \frac{3}{2}e^{2x}\sin(3x)dx$$

Use integration by parts again.

$$u = \sin(3x) \qquad du = 3\cos(3x)$$
$$v = \frac{1}{2}e^{2x} \qquad dv = e^{2x}dx$$

$$\int e^{2x}\cos(3x)dx = \frac{1}{2}e^{2x}\cos(3x) + \frac{3}{2}\left[\frac{1}{2}e^{2x}\sin(3x) - \frac{3}{2}\int e^{2x}\cos(3x)dx\right]$$
$$= \frac{1}{2}e^{2x}\cos(3x) + \frac{3}{4}e^{2x}\sin(3x) - \frac{9}{4}\int e^{2x}\cos(3x)dx$$

Let $I = \int e^{2x}\cos(3x)dx$

$$= \frac{1}{2}e^{2x}\cos(3x) + \frac{3}{4}e^{2x}\sin(3x) - \frac{9}{4}I + C_1.$$

Solve for I and rename the constant of integration as C.

$$\int e^{2x}\cos(3x)dx = \frac{e^{2x}(2\cos(3x) + 3\sin(3x))}{13} + C.$$

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2. Evaluate using integration by parts or substitution.

$$\int x^2 e^{14x} dx$$

Use the integration by parts formula, given below, to integrate. In some cases it is necessary to apply the formula more than once to integrate the expression completely.

$$\int u\,dv = uv - \int v\,du$$

2. answer

Evaluate using integration by parts or substitution.

$$\int x^2 e^{14x} dx$$

Use the integration by parts formula, given below, to integrate. In some cases it is necessary to apply the formula more than once to integrate the expression completely.

$$\int u\, dv = uv - \int v\, du$$

The integral $\int x^2 e^{14x} dx$ is in the form $\int u\, dv$. Determine choices for u and dv that will make the integration easier to solve.

Since the derivative of an exponential function is an exponential function, the choice $u = e^{14x}$ would not simplify the integration.

Using $u = x^2$, and $dv = e^{14x} dx$, integrate by parts. First find du and v.

$$u = x^2 \qquad dv = e^{14x} dx$$

$$du = 2x\, dx \qquad v = \frac{1}{14} e^{14x}$$

Calculate uv. Substitute x^2 for u, $\frac{1}{14} e^{14x}$ for v, and simplify.

$$uv = x^2\left(\frac{1}{14} e^{14x}\right)$$

$$= \frac{1}{14} x^2 e^{14x}$$

Next calculate vdu. Substitute $\frac{1}{14} e^{14x}$ for v, 2x dx for du, and simplify.

$$vdu = 2x\left(\frac{1}{14} e^{14x}\right) dx$$

2. answer cont.

$$= \frac{1}{7} x\, e^{14x} dx$$

Substitute the expressions into the formula.

$$\int u\, dv = uv - \int v\, du$$
$$= \frac{1}{14} x^2 e^{14x} - \frac{1}{7} \int x\, e^{14x} dx$$

The integral on the right, $\int x\, e^{14x} dx$, is still too complicated to integrate directly. Apply integration by parts again to evaluate the integral.

Determine choices for u and dv will make the integral, $\int x\, e^{14x} dx$, easier to solve.

Using $u = x$, and $dv = e^{14x} dx$, integrate by parts. Find du and v.

$$u = x \qquad dv = e^{14x} dx$$
$$du = dx \qquad v = \frac{1}{14} e^{14x}$$

Substitute the values for uv and vdu into the integration by parts formula and simplify.

$$\int u\, dv = uv - \int v\, du$$
$$= \frac{1}{14} x\, e^{14x} - \frac{1}{14} \int e^{14x} dx$$

The integral on the right, $\int e^{14x} dx$, can now be integrated directly. This is the same value that has been used for dv all along. Determine the value for the integral.

$$\int e^{14x} dx = \frac{1}{14} e^{14x}$$

The second integration by parts gives us the following.

$$\int x\, e^{14x} dx = \frac{1}{14} x\, e^{14x} - \frac{1}{14} \int e^{14x} dx$$
$$= \frac{1}{14} x\, e^{14x} - \frac{1}{14} \left(\frac{1}{14} e^{14x} \right)$$

Substitute the value for $\int x\, e^{14x} dx$ into the formula.

2. answer cont.

$$\int u\,dv = uv - \int v\,du$$
$$= \frac{1}{14}x^2 e^{14x} - \frac{1}{7}\int x e^{14x}dx$$
$$= \frac{1}{14}x^2 e^{14x} - \frac{1}{7}\left(\frac{1}{14}x e^{14x} - \frac{1}{14}\left(\frac{1}{14}e^{14x}\right)\right)$$

Simplify.

$$\int x^2 e^{14x}dx = \frac{1}{14}x^2 e^{14x} - \frac{1}{7}\left(\frac{1}{14}x e^{14x} - \frac{1}{14}\left(\frac{1}{14}e^{14x}\right)\right)$$
$$= \frac{1}{14}x^2 e^{14x} - \frac{1}{98}x e^{14x} + \frac{1}{1372}e^{14x} + C$$

Therefore, $\int x^2 e^{14x}dx = \frac{1}{14}x^2 e^{14x} - \frac{1}{98}x e^{14x} + \frac{1}{1372}e^{14x} + C.$

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3. Find the indefinite integral.

$$\int 4x \ln(2x)\,dx$$

Use integration by parts to find $\int 4x \ln(2x)\,dx$.

3. answer

Find the indefinite integral.

$$\int 4x \ln(2x)dx$$

Use integration by parts to find $\int 4x \ln(2x)dx$.

Use integration by parts to find $\int 4x \ln(2x)dx$.

With $u = \ln(2x)$ and $dv = 4x\ dx$, then $du = \frac{1}{x}dx$ and $v = 2x^2$.

Apply integration by parts.

$$\int u\ dv = uv - \int v\ du$$

$$\begin{aligned} \int 4x \ln(2x)dx &= 2x^2 \ln(2x) - \int 2x^2 \frac{1}{x}dx \\ &= 2x^2 \ln(2x) - \int 2x\ dx \\ &= 2x^2 \ln(2x) - x^2 + C \end{aligned}$$

Thus, $\int 4x \ln(2x)dx = 2x^2 \ln(2x) - x^2 + C$.

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4. Solve the initial value problem.

$$\frac{dy}{dx} = (x+6)\,\mathbf{sin}\,x \text{ and } y = 1 \text{ when } x = 0$$

To solve this initial value problem, first find the antiderivative of $\frac{dy}{dx}$.

$$y = \int \frac{dy}{dx}\,dx$$

$$= \int (x+6)\,\mathbf{sin}\,x\,dx$$

4. answer

Solve the initial value problem.

$$\frac{dy}{dx} = (x+6)\,\mathbf{sin}\,x \text{ and } y = 1 \text{ when } x = 0$$

To solve this initial value problem, first find the antiderivative of $\frac{dy}{dx}$.

$$y = \int \frac{dy}{dx}\,dx$$

$$= \int (x+6)\,\mathbf{sin}\,x\,dx$$

Because $(x+6)\,\mathbf{sin}\,x$ is the product of two differentiable functions of x, use integration by parts. The general formula for integration by parts is shown below.

$$\int u\,dv = uv - \int v\,du$$

If $\int u\,dv = \int (x+6)\,\mathbf{sin}\,x\,dx$, determine good choices for u and dv.

Since the derivative of $\mathbf{sin}\,x$ is $\mathbf{cos}\,x$, the choice $u = \mathbf{sin}\,x$ would not simplify the integration.

Let $u = x + 6$ and $dv = \mathbf{sin}\,x\,dx$.

If $u = x + 6$, then $du = dx$. If $dv = \mathbf{sin}\,x\,dx$, determine v.

$$v = -\,\mathbf{cos}\,x$$

Now calculate uv. Substitute and multiply.

$$uv = -(x+6)\,\mathbf{cos}\,x$$

Next, calculate v du. Substitute and multiply.

$$v\,du = -\,\mathbf{cos}\,x\,dx$$

Substitute the expressions in the formula and integrate.

$$\int u\,dv = uv - \int v\,du$$

$$= -(x+6)\,\mathbf{cos}\,x + \int \mathbf{cos}\,x\,dx$$

$$= -(x+6)\,\mathbf{cos}\,x + \mathbf{sin}\,x + C$$

4. answer cont.

$$= -(x+6)\cos x + \sin x + C$$

Therefore, $y = -(x+6)\cos x + \sin x + C$

Now use the initial condition to determine the particular solution.

Substitute $y = 1$ and $x = 0$ into the equation found above.

$$\begin{aligned} y &= -(x+6)\cos x + \sin x + C \\ 1 &= -(0+6)\cos 0 + \sin 0 + C \end{aligned}$$

Simplify the expression on the right side.

$$\begin{aligned} 1 &= -(0+6)\cos 0 + \sin 0 + C \\ 1 &= C - 6 \end{aligned}$$

Now solve this for C.

$$\begin{aligned} C - 6 &= 1 \\ C &= 7 \end{aligned}$$

Finally, use this value of C to write the particular solution.

$$y = -(x+6)\cos x + \sin x + 7$$