Calculus Chap. 6 section 2 Fundamental theorem example	Name: ____________ Date: ____________ Period: ____________

1. Solve the initial value problem using the fundamental theorem. (The answer will contain a definite integral.)

$F'(x) = e^{\sin x}$ and $F(7) = 3$

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1. answer

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Note that the fundamental theorem of calculus states that if f is continuous on [a, b], then the function $F(x) = \int_a^x f(t)\,dt$ has a derivative at every point x in [a, b], and the following is true.

$$\frac{df}{dx} = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Use the fundamental theorem to find the general solution for F(x).

$$F(x) = \int_7^x e^{\sin t}\,dt + C$$

Now, apply the initial condition, $F(7) = 3$, to find C.

$$\begin{aligned} F(x) &= \int_7^x e^{\sin t}\,dt + C \\ F(7) &= \int_7^7 e^{\sin t}\,dt + C && \text{Substitute 7 for x.} \\ 3 &= 0 + C && \text{Substitute 3 for F(7) and evaluate the integral.} \\ 3 &= C && \text{Solve for C.} \end{aligned}$$

Thus, the solution of the initial value problem is $F(x) = \int_7^x e^{\sin t}\,dt + 3$.