Calculus
Chap. 5 section 5
Definite Integrals & fundamental theorem cont.

Name: ______________________
Date: ______________________
Period: ______________________

1. Express the solution of the initial value problem in terms of an integral.

$\frac{dy}{dx} = \sin x,\ y(2) = 3$

Find the function $y = F(x)$ with derivative $f(x) = \frac{dy}{dx} = \sin x$ that satisfies the condition that $F(2) = 3$.

1. answer

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According to the first part of the fundamental theorem of calculus, if f is continuous on [a,b] then $F(x) = \int_a^x f(t)dt$ is continuous on [a,b] and differentiable on (a,b) and the following equation is true.

$$F'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$$

Since $F(2) = 3$, then $F(2) = \int_2^2 \sin t\, dt + 3 = 3$ because $\int_2^2 \sin t\, dt = 0$.

Thus, the solution of the initial value problem $\frac{dy}{dx} = \sin x,\ y(2) = 3$ in terms of an integral with variable t is $y = \int_2^x \sin t\, dt + 3$.

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Chap. 5 section 1
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2. Construct a function of the form $y = \int_a^x f(t)\,dt + C$ that satisfies the given conditions.

$\frac{dy}{dx} = \ln(\sin x + 8)$, and $y = 4$ when $x = 5$.

First find a function y, whose derivative is $\frac{dy}{dx} = \ln(\sin x + 8)$, using the fundamental theorem of calculus, $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$.

2. answer

Construct a function of the form $y = \int_a^x f(t)\,dt + C$ that satisfies the given conditions.

$$\frac{dy}{dx} = \ln(\sin x + 8), \text{ and } y = 4 \text{ when } x = 5.$$

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Since $f(x) = \ln(\sin x + 8)$, find f(t).

$$f(t) = \ln(\sin t + 8)$$

To get a function y with the derivative $\ln(\sin x + 8)$, substitute $\ln(\sin t + 8)$ for f(t) and 5 for a, since 5 involves fewer calculations in the next step.

$$y = \int_a^x f(t)\,dt + C$$

$$= \int_5^x \ln(\sin t + 8)\,dt + C$$

Use the fact that y = 4 when x = 5 to find C.

$$4 = \int_5^5 \ln(\sin t + 8)\,dt + C$$

$$= 0 + C$$

Therefore, C = 4.

Hence, the function $y = \int_5^x \ln(\sin t + 8)\,dt + 4$ satisfies the given conditions.

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3. Construct a function of the form $y = \int_a^x f(t)\,dt + C$ that satisfies the given conditions.

$\frac{dy}{dx} = \cos^2 6x$, and $y = -5$ when $x = 4$.

The Fundamental Theorem of Calculus

If f is continuous on [a,b], then the function

$$F(x) = \int_a^x f(t)\,dt$$

has a derivative at every point x in [a,b], and

$$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

3. answer

Construct a function of the form $y = \int_a^x f(t)\,dt + C$ that satisfies the given conditions.

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$$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

The solution is to be of the form $y = \int_a^{x} f(t)\,dt + C$. The upper limit of integration is x. The lower limit of integration will be the given value of x. Thus, $a = 4$.

Now determine the integrand. The fundamental theorem of calculus says that $\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$. Therefore, $y = \int_4^x \cos^2 6t\,dt + C$.

Next determine the value of the arbitrary constant C using the condition that $y = -5$ when $x = 4$.

$$-5 = \int_4^4 \cos^2 6t\,dt + C$$
$$-5 = 0 + C$$
$$-5 = C$$

The equation $y = \int_4^x \cos^2 6t\,dt - 5$ meets the given conditions.

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4. Evaluate the integral using Part 2 of the Fundamental Theorem of Calculus.

$$\int_0^1 (7x^2 + 7\sqrt{x})\, dx$$

Part 2 of the Fundamental Theorem of Calculus states that $\int_a^b f(x)\, dx = F(b) - F(a)$.

4. answer

Evaluate the integral using Part 2 of the Fundamental Theorem of Calculus.

$$\int_0^1 (7x^2 + 7\sqrt{x})\ dx$$

Part 2 of the Fundamental Theorem of Calculus states that $\int_a^b f(x)\ dx = F(b) - F(a)$.

First find an antiderivative F(x) of $7x^2 + 7\sqrt{x}$ by finding an antiderivative for each of the terms and then simplify.

$$F(x) = \frac{7x^3}{3} + \frac{14x^{3/2}}{3}$$

Evaluate over the limits of x and simplify.

$$\int_0^1 (7x^2 + 7\sqrt{x})\ dx = \left[\frac{7x^3}{3} + \frac{14x^{3/2}}{3}\right]_0^1$$

$$= F(1) - F(0)$$

$$= (7) - (0)$$

$$\int_0^1 (7x^2 + 7\sqrt{x})\ dx = 7$$

Support your answer using NINT on the calculator.

$$\text{NINT}\left(7x^2 + 7\sqrt{x}, x, 0, 1\right) = 7$$

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5.

Evaluate the integral $\int_1^8 x^{-(7/3)}\,dx$.

According to the second part of the fundamental theorem of calculus, if f is continuous at every point of [a,b] and F is any antiderivative of f on [a,b], then the definite integral can be evaluated using the following formula.

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

5. answer

Evaluate the integral $\int_1^8 x^{-(7/3)}dx$.

According to the second part of the fundamental theorem of calculus, if f is continuous at every point of [a,b] and F is any antiderivative of f on [a,b], then the definite integral can be evaluated using the following formula.

$$\int_a^b f(x)dx = F(b) - F(a).$$

The general antiderivative of $f(x) = x^{-7/3}$ is $F(x) = -\frac{3}{4}x^{-4/3} + C$.

Because of the subtraction, a constant in F(x) will not affect the value of F(b) − F(a). Thus, there is no need to include the constant C.

$$\begin{aligned}\int_1^8 x^{-(7/3)}dx &= F(8) - F(1)\\ &= -\frac{3}{4}x^{-4/3}\Big]_1^8\end{aligned}$$

Next, calculate F(8).

$$\begin{aligned}F(8) &= -\frac{3}{4}(8)^{-4/3}\\ &= -\frac{3}{64}\end{aligned}$$

Calculate F(1).

$$\begin{aligned}F(1) &= -\frac{3}{4}(1)^{-4/3}\\ &= -\frac{3}{4}\end{aligned}$$

Finally, evaluate the integral.

$$\begin{aligned}\int_1^8 x^{-(7/3)}dx &= F(8) - F(1)\\ &= -\frac{3}{64} - \left(-\frac{3}{4}\right)\\ &= \frac{45}{64}\end{aligned}$$