Calculus
Chap. 5 section 4
Definite Integral & fundamental theorem

Name: ______________________
Date: ______________________
Period: ______________________

1. Find $\frac{dy}{dx}$.

$$y = \int_6^x (t^2 - t)^7 dt$$

The Fundamental Theorem of Calculus

If f is continuous on [a,b], then the function

$$F(x) = \int_a^x f(t)\, dt$$

has a derivative at every point x in [a,b], and

$$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)\, dt = f(x).$$

1. answer

Find $\frac{dy}{dx}$.

$$y = \int_6^x (t^2 - t)^7 dt$$

The Fundamental Theorem of Calculus

If f is continuous on [a,b], then the function

$$F(x) = \int_a^x f(t)\,dt$$

has a derivative at every point x in [a,b], and

$$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

Since the upper limit of integration is x and the lower limit of integration is a constant, $y = \int_6^x (t^2 - t)^7 dt$ is in the correct form to use the fundamental theorem of calculus to find the integral.

Use the fundamental theorem of calculus to find $\frac{dy}{dx}$.

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\int_6^x (t^2 - t)^7 dt \\ &= (x^2 - x)^7\end{aligned}$$

Name: ______________________
Date: ______________________
Period: ______________________

2.

Find $\frac{dy}{dx}$.

$$y = \int_{-2\pi}^{x} \frac{6 - \sin t}{4 + \cos t}\, dt$$

The Fundamental Theorem of Calculus

If f is continuous on [a,b], then the function

$$F(x) = \int_{a}^{x} f(t)\, dt$$

has a derivative at every point x in [a,b], and

$$\frac{dF}{dx} = \frac{d}{dx}\int_{a}^{x} f(t)\, dt = f(x).$$

2. answer

Find $\frac{dy}{dx}$.

$$y = \int_{-2\pi}^{x} \frac{6 - \sin t}{4 + \cos t}\, dt$$

The Fundamental Theorem of Calculus

If f is continuous on [a,b], then the function

$$F(x) = \int_{a}^{x} f(t)\, dt$$

has a derivative at every point x in [a,b], and

$$\frac{dF}{dx} = \frac{d}{dx}\int_{a}^{x} f(t)\, dt = f(x).$$

Since the upper limit of integration is x and the lower limit of integration is a constant, $y = \int_{-2\pi}^{x} \frac{6 - \sin t}{4 + \cos t}\, dt$ is in the correct form to use the fundamental theorem of calculus to find the derivative.

Use the fundamental theorem of calculus to find $\frac{dy}{dx}$.

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\int_{-2\pi}^{x} \frac{6 - \sin t}{4 + \cos t}\, dt \\ &= \frac{6 - \sin x}{4 + \cos x}\end{aligned}$$

Calculus	Name: ______
Chap. 5 section 4	Date: ______
Definite Integrals & fundamental theorem	Period: ______

3.

Find the derivative of $\frac{d}{dx}\int_0^{x^5} e^{-6t}\,dt$

a. by evaluating the integral and differentiating the result.
b. by differentiating the integral directly.

---

a. To find the derivative by evaluating the integral and differentiating the result, begin by using $\int_a^b f(x)\,dx = F(b) - F(a)$. In this case, $\int_a^b f(x)\,dx$ is $\int_0^{x^5} e^{-6t}\,dt$.

3. answer

Find the derivative of $\frac{d}{dx}\int_0^{x^5} e^{-6t}\,dt$

a. by evaluating the integral and differentiating the result.
b. by differentiating the integral directly.

---

a. To find the derivative by evaluating the integral and differentiating the result, begin by using $\int_a^b f(x)\,dx = F(b) - F(a)$. In this case, $\int_a^b f(x)\,dx$ is $\int_0^{x^5} e^{-6t}\,dt$.

To calculate the definite integral of f over [a,b], find an antiderivative F of f, and calculate the number $\int_a^b f(x)\,dx = F(b) - F(a)$. The usual notation for $F(b) - F(a)$ is $\left[F(x)\right]_a^b$.

Begin by finding F, the antiderivative of the function e^{-6t}.

$$\int_0^{x^5} e^{-6t}\,dt = \left[\frac{-e^{-6t}}{6}\right]_0^{x^5}$$

Now evaluate the definite integral using $F = \frac{-e^{-6t}}{6}$. Substitute in the upper and lower limits of the integral.

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$\int_0^{x^5} e^{-6t}\,dt = \left[-\frac{e^{-6x^5}}{6}\right] - \left[-\frac{e^{-6\cdot 0}}{6}\right]$$

Simplify to find $\int_0^{x^5} e^{-6t}\,dt$.

$$\int_0^{x^5} e^{-6t}\,dt = \left[-\frac{e^{-6x^5}}{6}\right] - \left[-\frac{e^{-6\cdot 0}}{6}\right] = \frac{-e^{-6x^5}}{6} + \frac{1}{6}$$

Substitute the result back into the original expression.

$$\frac{d}{dx}\int_0^{x^5} e^{-6t}\,dt = \frac{d}{dx}\left(\frac{-e^{-6x^5}}{6} + \frac{1}{6}\right)$$

3. answer cont.

Now differentiate $\frac{-e^{-6x^5}}{6}+\frac{1}{6}$ with respect to x to obtain the derivative. Since the derivative of a contant is 0, find $\frac{d}{dx}\left(-\frac{e^{-6x^5}}{6}\right)$. Use the chain rule, $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$, to differentiate. Let $u=-6x^5$.

$$\frac{du}{dx}=-30x^4$$

Use the fact that $\frac{d}{du}(a e^u)=a e^u$ to find the derivative.

$$\frac{d}{dx}\left(-\frac{e^{-6x^5}}{6}\right)=-e^u\cdot\frac{du}{dx}=5x^4e^{-6x^5}$$

b. To find the derivative by differentiating the integral directly, use $F'(x)=\frac{d}{dx}\int_a^x f(t)\,dt=f(x)$. For this problem, let $y=\int_0^{x^5} e^{-6t}\,dt$.

The upper limit of integration is not x, but x^5. This makes y a composite of the two functions, $y=\int_0^u e^{-6t}\,dt$ and $u=x^5$. Therefore, apply the chain rule, $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$, when finding $\frac{dy}{dx}$.

First find the value of $\frac{du}{dx}$.

$$\frac{du}{dx}=5x^4$$

The derivative is given below.

$$\begin{aligned}\frac{dy}{dx}&=\left(\frac{d}{du}\int_0^u e^{-6t}\,dt\right)\cdot\frac{du}{dx}\\&=e^{-6u}\cdot\frac{du}{dx}\\&=5x^4e^{-6x^5}\qquad (u=x^5)\end{aligned}$$

Calculus Chap. 5 section 4 Definite Integrals & fundamental theorem	Name: ______________________ Date: ______________________ Period: ______________________

4. Find $\frac{dy}{dx}$.

$$y = \int_x^8 \ln(6 + t^3)\, dt$$

The Fundamental Theorem of Calculus

If f is continuous on [a,b], then the function

$$F(x) = \int_a^x f(t)\, dt$$

has a derivative at every point x in [a,b], and

$$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)\, dt = f(x).$$

4. answer

Find $\frac{dy}{dx}$.

$$y = \int_x^8 \ln(6+t^3)\,dt$$

The Fundamental Theorem of Calculus

If f is continuous on [a,b], then the function

$$F(x) = \int_a^x f(t)\,dt$$

has a derivative at every point x in [a,b], and

$$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

Since the integral has a variable in the lower limit of integration, $y = \int_x^8 \ln(6+t^3)\,dt$ is not in the correct form to use the fundamental theorem of calculus to find the derivative.

Use the rules for integrals to set up $y = \int_x^8 \ln(6+t^3)\,dt$ for the fundamental theorem of calculus. The rule for order of integration states that $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$. Therefore, $y = -\int_8^x \ln(6+t^3)\,dt$.

Use the fundamental theorem of calculus to find $\frac{dy}{dx}$.

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(-\int_8^x \ln(6+t^3)\,dt\right)\\ &= -\ln(6+x^3)\end{aligned}$$

Calculus Chap. 5 section 4 Definite Integrals & fundamental theorem	Name: ________________ Date: ________________ Period: ________________

5.

Find $\frac{dy}{dx}$ for $y = \int_{10\sqrt{x}}^{0} \mathbf{sin}\,(t^2)\,dt$.

Differentiate the integral directly using $F'(x) = \frac{d}{dx}\int_{a}^{x} f(t)\,dt = f(x)$.

5. answer

Find $\frac{dy}{dx}$ for $y = \int_{10\sqrt{x}}^{0} \sin(t^2)\, dt$.

Differentiate the integral directly using $F'(x) = \frac{d}{dx}\int_a^x f(t)\, dt = f(x)$.

Notice that y is currently in the form $\int_x^a f(t)\, dt$. In order to use the rule given above, use the rule $\int_b^a f(x)\, dx = -\int_a^b f(x)\, dx$ to get the x-term in the upper limit.

$$\int_{10\sqrt{x}}^{0} \sin(t^2)\, dt = -\int_0^{10\sqrt{x}} \sin(t^2)\, dt$$

The upper limit of integration is not x, but $10\sqrt{x}$. This makes y a composite of the two functions, $y = -\int_0^u \sin(t^2)\, dt$ and $u = 10\sqrt{x}$. Therefore, apply the chain rule, $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, when finding $\frac{dy}{dx}$.

First find $\frac{du}{dx}$.

$$\frac{du}{dx} = \frac{d}{dx}(10\sqrt{x}) = 5x^{-1/2}$$

Next find $\frac{dy}{du}$.

$$\frac{d}{du}\left(-\int_0^u \sin(t^2)\, dt\right) = -\sin u^2$$

The derivative is shown below.

$$\begin{aligned} \frac{dy}{dx} &= -\sin u^2 \cdot \frac{du}{dx} \\ &= -5x^{-1/2}\sin(100x) \qquad (u = 10\sqrt{x}) \end{aligned}$$