Calculus
Chap. 5 section 3
Definite Integrals & antiderivatives cont.

Name: ______________________
Date: ______________________
Period: ______________________

1. Interpret the integrand as the rate of change of a quantity and evaluate the integral using the antiderivative of the quantity.

$$\int_0^{\pi} 6\sin x\,dx$$

To evaluate the integral, start by finding the antiderivative of $6\sin x$. That is, find the function whose derivative is $6\sin x$. Recall that $\frac{d}{dx}\cos x = -\sin x$. Therefore, the antiderivative of $6\sin x$ is $F(x) = -6\cos x + C$, where C is an arbitrary constant.

2. Evaluate the integral $\int_2^5 9u^2\,du$.

Apply the Constant Multiple rule.

$$\int_2^5 9u^2\,du = 9\int_2^5 u^2\,du$$

1. answer

Interpret the integrand as the rate of change of a quantity and evaluate the integral using the antiderivative of the quantity.

$$\int_0^{\pi} 6 \sin\ x\, dx$$

To evaluate the integral, start by finding the antiderivative of $6 \sin\ x$. That is, find the function whose derivative is $6 \sin\ x$. Recall that $\frac{d}{dx} \cos x = -\sin x$. Therefore, the antiderivative of $6 \sin\ x$ is $F(x) = -6 \cos\ x + C$, where C is an arbitrary constant.

The definite integral $\int_a^x f(t)\, dt = F(x) - F(a)$, where F is the antiderivative of f.

Thus, $\int_0^{\pi} 6 \sin\ x\, dx = -6 \cos\ \pi - (-6 \cos\ 0)$. Evaluate the integral.

$$\begin{aligned}\int_0^{\pi} 6 \sin\ x\, dx &= -6 \cos\ \pi - (-6 \cos\ 0) \\ &= 6 - (-6) \\ &= 12\end{aligned}$$

2. answer

Evaluate the integral $\int_2^5 9u^2 du$.

Apply the Constant Multiple rule.

$$\int_2^5 9u^2 du = 9\int_2^5 u^2 du$$

Now, in general $\int_a^b x^2 dx = \frac{b^3}{3} - \frac{a^3}{3}$.

So, $9\int_2^5 u^2 du = 9\left(\frac{5^3}{3} - \frac{2^3}{3}\right)$.

Therefore, $9\int_2^5 u^2 du = 351$.

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3. Evaluate the integral $\int_8^4 3\ dx$.

For any constant c, $\int_a^b c\ dx = c(b - a)$.

So, $\int_8^4 3\ dx = 3(4 - 8)$.

Thus, $\int_8^4 3\ dx = -12$.

4. Find the average value of the function on the interval, using antiderivatives to compute the integral.

$y = 6\ \mathbf{sin}\ x,\ \ [0,\pi]$

The average (mean) value theorem states that if f is integrable on [a,b], its average (mean) value on [a,b] is $av(f) = \frac{1}{b-a}\int_a^b f(x)\ dx$.

3. answer cont.

Evaluate the integral $\int_8^4 3\,dx$.

For any constant c, $\int_a^b c\,dx = c(b-a)$.

So, $\int_8^4 3\,dx = 3(4-8)$.

Thus, $\int_8^4 3\,dx = -12$.

4. answer

Find the average value of the function on the interval, using antiderivatives to compute the integral.

$y = 6\sin x,\ [0,\pi]$

The average (mean) value theorem states that if f is integrable on [a,b], its average (mean) value on [a,b] is $av(f) = \frac{1}{b-a}\int_a^b f(x)\,dx$.

To find the average value, start by finding the antiderivative of $6\sin x$. That is, find the function whose derivative is $6\sin x$. Recall that $\frac{d}{dx}\cos x = -\sin x$. Therefore, the antiderivative of $6\sin x$ is $F(x) = -6\cos x + C$, where C is an arbitrary constant.

The definite integral $\int_a^x f(t)\,dt = F(x) - F(a)$, where F is the antiderivative of f.

Thus, $\int_0^\pi 6\sin x\,dx = -6\cos(\pi) - (-6\cos(0))$. Evaluate the integral.

$$\begin{aligned}\int_0^\pi 6\sin x\,dx &= -6\cos(\pi) - (-6\cos(0))\\ &= 6-(-6)\\ &= 12\end{aligned}$$

Now find the average value of the function on the given interval using the average value formula.

$$\begin{aligned}av(y) &= \frac{1}{(\pi)-(0)}\int_0^\pi 6\sin x\,dx\\ &= \frac{12}{\pi}\end{aligned}$$

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3.

3. answer