Calculus
Chap. 5 section 3
Definite Integral & antiderivatives

Name: ______________________________
Date: ______________________________
Period: ______________________________

**1.** The functions f and g are integrable and $\int_2^4 f(x)dx = -5$, $\int_2^7 f(x)dx = 3$, and $\int_2^7 g(x)dx = 4$. Find the values of the following definite integrals.

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Considering the definite integral as the area under the graph of a function over an interval [a,b], if the interval has zero width, the area is zero and the value of the definite integral is zero. This gives to the Zero Width Interval rule: $\int_a^a f(x)dx = 0$.

So, $\int_4^4 f(x)dx = 0$.

## 1. answer

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So, $\int_4^4 f(x)dx = 0$.

The Order of Integration rule states that $\int_a^b f(x)dx = -\int_b^a f(x)dx$.

So, $\int_7^2 g(x)dx = -4$.

There is a Constant Multiple rule for definite integrals as there is for limits.

So, $\int_2^7 2g(x)dx = 2\int_2^7 g(x)dx = 2(4) = 8$.

Again considering the definite integral as the area under the graph of a function over an interval [a,b]; for c such that $a < c < b$, the area over [c,b] is equal to the area over [a,b] minus the area over [a,c].

Thus, $\int_4^7 f(x)dx = \int_2^7 f(x)dx - \int_2^4 f(x)dx = 3 - (-5) = 8$.

There is a Sum/Difference rule for definite integrals as there is for limits.

So, $\int_2^7 [g(x) - f(x)]dx = \int_2^7 g(x)dx - \int_2^7 f(x)dx = 4 - 3 = 1$.

Applying both the Sum/Difference and the Constant Multiple rules to $\int_2^7 [4g(x) - f(x)]dx$, the value of the definite integral is 13.

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**2.** Find the average value of the function $f(x) = x^2 - 5$ on $[0, \sqrt{10}]$.

The average value of f(x) on [a,b] is defined as $\frac{1}{b-a}\int_a^b f(x)dx$.

## 2. answer

Find the average value of the function $f(x)=x^2-5$ on $[0,\sqrt{10}]$.

The average value of f(x) on [a,b] is defined as $\frac{1}{b-a}\int_a^b f(x)dx$.

So, the average value of $f(x)=x^2-5$ on $[0,\sqrt{10}]$ is $\frac{1}{\sqrt{10}-0}\int_0^{\sqrt{10}}(x^2-5)dx$.

Applying the Difference and Constant Multiple rules, $\frac{1}{\sqrt{10}-0}\int_a^b[x^2-5]dx=\frac{1}{\sqrt{10}}\left(\int_0^{\sqrt{10}}x^2dx-5\int_0^{\sqrt{10}}dx\right)$.

$$\int_0^{\sqrt{10}}x^2dx=\frac{10^{3/2}}{3}$$

Since $\int_0^{\sqrt{10}}dx=\sqrt{10}$, the following is true.

$$\begin{aligned}\frac{1}{\sqrt{10}}\left(\int_0^{\sqrt{10}}x^2dx-5\int_0^{\sqrt{10}}dx\right) &= \frac{1}{\sqrt{10}}\left(\frac{10^{3/2}}{3}-5\sqrt{10}\right)\\ &= \frac{10}{3}-5\\ &= -\frac{5}{3}\end{aligned}$$

So, the average value of $f(x)=x^2-5$ on $[0,\sqrt{10}]$ is $-\frac{5}{3}$.

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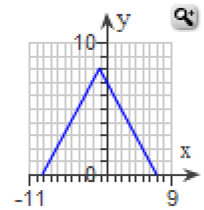
**3.** Find the average value of the function on the interval without integrating, by appealing to the geometry of the region between the graph and the x-axis.

$$f(x) = \begin{cases} x+9 & -9 \le x \le -1 \\ -x+7 & -1 < x \le 7 \end{cases} \quad \text{on } [-9,7]$$




The average (mean) value theorem states that if f is integrable on [a,b], its average (mean) value on [a,b] is $av(f) = \frac{1}{b-a}\int_a^b f(x)\,dx$.

## 3. answer

Find the average value of the function on the interval without integrating, by appealing to the geometry of the region between the graph and the x-axis.

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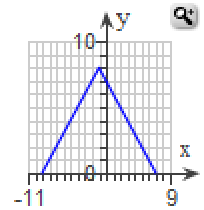


The average (mean) value theorem states that if f is integrable on [a,b], its average (mean) value on [a,b] is $av(f) = \frac{1}{b-a}\int_a^b f(x)\,dx$.

In order to find the average value of f(x) on the interval $[-9,7]$, first find the area between the graph and the x-axis using geometry. Note the region is in the shape of a triangle.

The base of the triangle has a length equal to the distance between the two x-intercepts, that is the distance between the endpoints of the interval $[-9,7]$.

The length of the base is $7-(-9)=16$

The height of the triangle is equal to the distance from the base of the triangle to the vertex opposite the base. This distance is equal to the value of the function when $x=-1$.

$$\begin{aligned} f(-1) &= -1+9 \\ &= 8 \end{aligned}$$

The height is 8.

The area, A, of the triangle is given by the formula $A = \frac{1}{2}bh$.

$$\begin{aligned} A &= \frac{1}{2}(16)(8) \\ &= 64 \end{aligned}$$

Thus, $\int_{-9}^{7} f(x)\,dx = 64$.

Now use the average (mean) value formula to find the average value of the function on the interval $[-9,7]$.

$$\begin{aligned} av(f) &= \frac{1}{7-(-9)}\int_{-9}^{7} f(x)\,dx \\ &= \frac{1}{16}(64) \\ &= 4 \end{aligned}$$

The average value of the function is 4.

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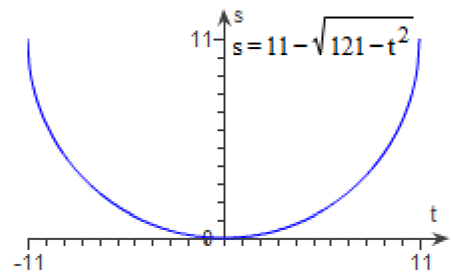
**4.** Find the average value of the function on the interval without integrating, by appealing to the geometry of the region between the graph and the x-axis.

$$f(t) = 11 - \sqrt{121 - t^2}, \quad [-11, 11]$$

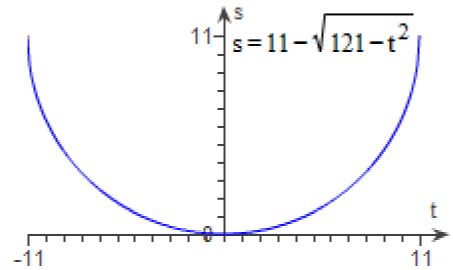



The average (mean) value theorem states that if f is integrable on [a,b], its average (mean) value on [a,b] is $av(f) = \frac{1}{b-a}\int_a^b f(t)\,dt$.

## 4. answer

Find the average value of the function on the interval without integrating, by appealing to the geometry of the region between the graph and the x-axis.

$$f(t) = 11 - \sqrt{121 - t^2}, \quad [-11, 11]$$




The average (mean) value theorem states that if f is integrable on [a,b], its average (mean) value on [a,b] is $av(f) = \frac{1}{b-a}\int_a^b f(t)\,dt$.

value on [a,b] is $av(f) = \frac{1}{b-a}\int_a^b f(t)\,dt$.

Determine the value of $\int_a^b f(t)\,dt$ by appealing to the geometry of the region between the graph and the t-axis. Notice that the region between the graph and the t-axis is the difference between the area of the rectangle that bounds the semicircle and the area of the semicircle.

The area of the rectangle is equal to LW.

$$(22)(11) = 242$$

The area of the semicircle is equal to $\frac{1}{2}\pi r^2$.

$$\frac{1}{2}\pi(11)^2 = \frac{121\pi}{2}$$

Now find the difference of the areas to determine $\int_a^b f(t)\,dt$, the region between the graph and the t-axis.

$$\int_a^b f(t)\,dt = \int_{-11}^{11} f(t)\,dt$$
$$= 242 - \frac{121\pi}{2}$$

Substitute the expression for $\int_a^b f(t)\,dt$ into the formula for the average value of a function and simplify.

**4. answer cont.**

$$\begin{aligned} av(f) &= \frac{1}{b-a}\int_a^b f(t)\,dt \\ &= \frac{1}{11-(-11)}\left(242-\frac{121\pi}{2}\right) \\ &= \frac{44-11\pi}{4} \end{aligned}$$

Therefore, the average value of the function $f(t) = 11 - \sqrt{121 - t^2}$ on the interval $[-11, 11]$ is $\frac{44-11\pi}{4}$.