Calculus	Name: ______________________
Chap. 5 section 2	Date: ______________________
Definite Integral	Period: ______________________

1. Evaluate the integral $\int_{-3}^{4} 8\,dx$.

For any constant c, $\int_{a}^{b} c\,dx = c(b-a)$.

2. Graph the integrand, and use area to evaluate the definite integral $\int_{-6}^{8} (x+8)\,dx$.

The integrand is $f(x) = x + 8$.

1. answer

Evaluate the integral $\int_{-3}^{4} 8\ dx$.

For any constant c, $\int_{a}^{b} c\ dx = c(b-a)$.

Substitute the values of a, b, and c from the given integral into the integral of a constant and simplify.

$$\int_{-3}^{4} 8\ dx = 8(4-(-3))$$

$$= 8(7)$$

$$= 56$$

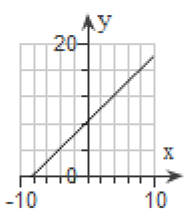
$$\int_{-3}^{4} 8\ dx = 56$$

2. answer

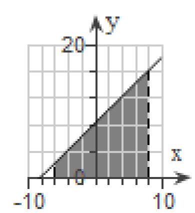
Graph the integrand, and use area to evaluate the definite integral $\int_{-6}^{8} (x+8)dx$.

The integrand is $f(x) = x + 8$.

The graph of the integrand, $f(x) = x + 8$ is shown below.



The area equivalent to the value of the definite integral is bounded by the graph, the x-axis, and the lines $x = -6$ and $x = 8$. The boundaries form a trapezoid with area equal to one half the product of the altitude times the sum of the bases.



The value of this area is 126.

Thus, $\int_{-6}^{8} (x+8)dx = 126$.

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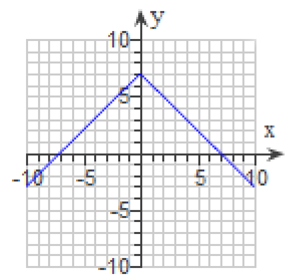
3. Use the graph of the integrand and areas to evaluate the integral.

$$\int_{-4}^{4} (7-|x|)\,dx$$

The first step to evaluating the integral $\int_{-4}^{4} (7-|x|)\,dx$ is to graph the integrand, $(7-|x|)$. The graph of the integrand is shown on the right.

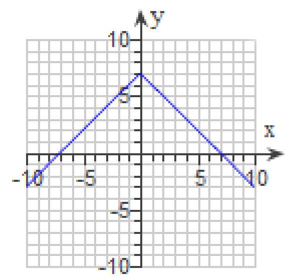


3. answer

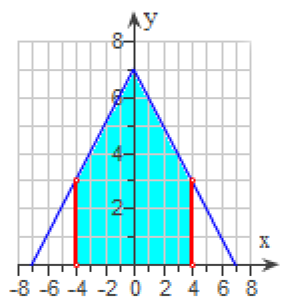
Use the graph of the integrand and areas to evaluate the integral.

$$\int_{-4}^{4} (7-|x|)\,dx$$

The first step to evaluating the integral $\int_{-4}^{4} (7-|x|)\,dx$ is to graph the integrand, $(7-|x|)$. The graph of the integrand is shown on the right.



$\int_{-4}^{4} (7-|x|)\,dx$ equals the area between the x-axis and the curve as x goes from -4 to 4. The graph on the right shows the area to be calculated.

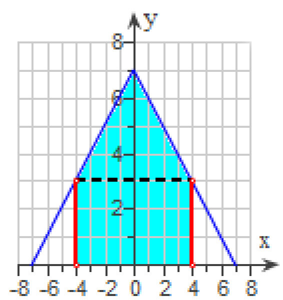


Notice that the area under the curve between $x=-4$ and $x=4$ can be split into two shapes, a rectangle and a triangle.

The area of a rectangle is given by $A = L \cdot W$.

$$A = 8 \cdot 3$$
$$= 24$$

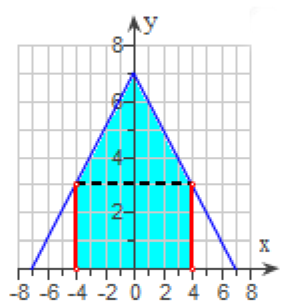
The area of the rectangle is 24.



The area of a triangle is given by $A = \frac{1}{2} b \cdot h$.

$$A = \frac{1}{2}(8)(4)$$
$$= 16$$

The area of the triangle is 16.



3. answer

To find the total area under the curve between $x = -4$ and $x = 4$ add the area of the rectangle to the area of the triangle.

$$\begin{aligned} \text{Area} &= 24 + 16 \\ &= 40 \end{aligned}$$

The area under the curve is 40.

Thus, $\int_{-4}^{4} (7 - |x|)\, dx = 40$.

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4. Use area to evaluate the integral $\int_0^b \frac{5}{6}x\,dx,\ b > 0.$

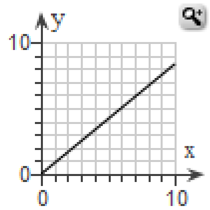
The integrand is $f(x) = \frac{5}{6}x.$

4. answer

Use area to evaluate the integral $\int_0^b \frac{5}{6}x\,dx$, $b > 0$.

The integrand is $f(x) = \frac{5}{6}x$.

The graph of the integrand, $f(x) = \frac{5}{6}x$ is shown below.



The lower boundary of the area is the line $y = 0$.

The area is bounded on the right by the line $x = b$.

Therefore, the area is bounded by the graph of the function $f(x) = \frac{5}{6}x$, the line $y = 0$, and the line $x = b$. Note that this is a triangle with area equal to one half the product of its base and height.

The value of this area is $\frac{5}{12}b^2$.

Thus, $\int_0^b \frac{5}{6}x\,dx = \frac{5}{12}b^2$.