Calculus
Chap. 5 section 1
Definite Integral—Finite Sums

Name: ____________________
Date: ____________________
Period: ____________________

1. A particle starts at $x = 0$ and moves along the x-axis with velocity $v(t) = t^2 + 6$ for time $t \geq 0$. Where is the particle at $t = 4$? Approximate the area under the curve using four rectangles of equal width and heights determined by the midpoints of the intervals.

Start by graphing v and partitioning the time interval [0,4] into subintervals of length Δt. First determine the width of the subintervals. To find the width of the subintervals, divide the width of the interval by the number of subintervals.

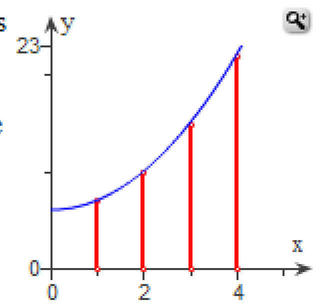
1. answer

A particle starts at $x = 0$ and moves along the x-axis with velocity $v(t) = t^2 + 6$ for time $t \ge 0$. Where is the particle at $t = 4$? Approximate the area under the curve using four rectangles of equal width and heights determined by the midpoints of the intervals.

Start by graphing v and partitioning the time interval [0,4] into subintervals of length Δt. First determine the width of the subintervals. To find the width of the subintervals, divide the width of the interval by the number of subintervals.

$$\Delta t = \frac{4-0}{4}$$
$$= 1$$

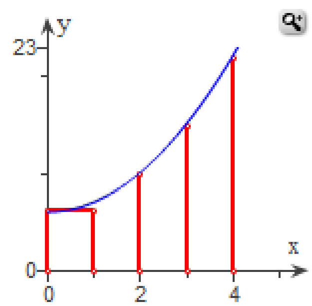
Notice that the region under the curve is partitioned into thin strips with bases of length 1 and curved tops. The area under the curve can be approximated by finding the area of a suitable rectangle. In this problem use the rectangle whose height is the y-coordinate of the function at the midpoint of the base.



$$m_1 = \frac{0+1}{2}$$
$$= 0.5$$

The height of the first rectangle is the value of the function at $t = 0.5$. Find the height of the first rectangle.

$$\text{height} = (0.5)^2 + 6$$
$$= 6.25$$



The area of a rectangle is equal to its length times its width. Therefore, the area of the first rectangle is 6.25.

$$\text{Area} = 1 \cdot 6.25$$
$$= 6.25$$

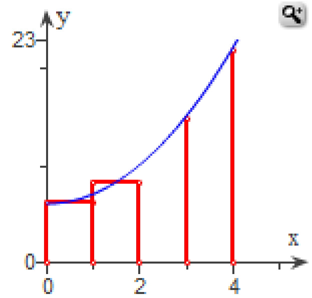
Now find the area of the second rectangle. Start by finding the midpoint of the second interval, [1,2].

$$m_2 = \frac{1+2}{2}$$
$$= 1.5$$

1. answer cont.

The height of the second rectangle is the value of the function at $t = 1.5$. Find the height of the second rectangle.

$$\text{height} = (1.5)^2 + 6$$
$$= 8.25$$



The area of the second rectangle is $1 \cdot 8.25 = 8.25$.

Continue in this manner to find the areas of the third and fourth rectangles.

For the third rectangle its midpoint is $m_3 = 2.5$ and its height is 12.25. Therefore, the area of the third rectangle is $1 \cdot 12.25 = 12.25$.

For the fourth rectangle its midpoint is $m_4 = 3.5$ and its height is 18.25. Therefore, the area of the fourth rectangle is $1 \cdot 18.25 = 18.25$.

To determine the total area under the curve add the areas.

$6.25 + 8.25 + 12.25 + 18.25 = 45$

Thus, the total distance the particle traveled was approximately 45 units.

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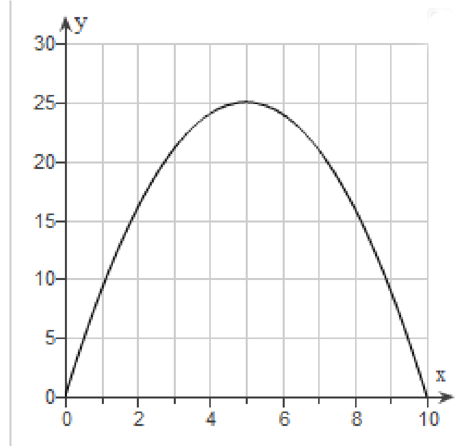
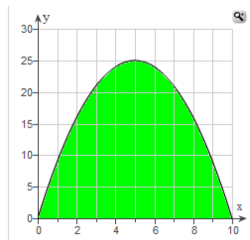
2. Sketch the region R enclosed between the graph of the function $y = 10x - x^2$ and the x-axis for $0 \le x \le 10$. Partition [0,10] into four subintervals and show the four rectangles that LRAM uses to approximate the area of R. Then compute the LRAM sum.

First, graph $y = 10x - x^2$.

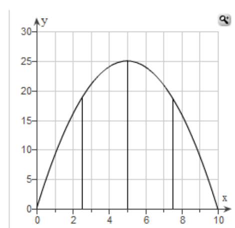


The region R is the area between the graph and the x-axis.

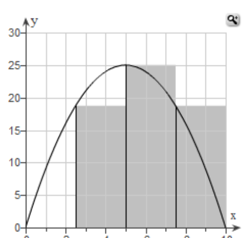
To approximate the area of the region R, divide the region into narrow vertical strips whose areas can be approximated by rectangles.



Partition [0,10] into four subintervals.
The width of each subinterval, Δx, is 2.5.



In LRAM, or the Left Rectangular Approximation Method, use rectangles whose height is the y-coordinate of the function at the left endpoint, x_i, of each subinterval.



The area of each rectangle is the rectangle's height, $f(x_i)$ or y, times its base width, or Δx.

$$\text{Area} = f(x_i) \cdot \Delta x$$

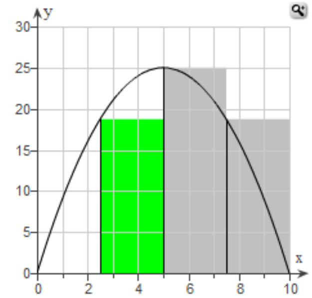
For the first subinterval, [0,2.5], the left endpoint is at $x = 0$, so the height of the rectangle in this subinterval is y at $x = 0$.

$$y = 10 \cdot 0 - 0^2$$

$$= 0$$

2. answer

Since the height of the first rectangle is 0, its area is also 0.

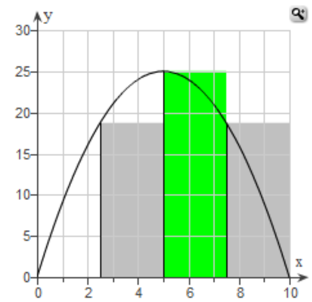


The height of the next rectangle, in the subinterval [2.5,5], is equal to y at x = 2.5.

$$y = 10 \cdot 2.5 - 2.5^2$$

$$= 18.75$$

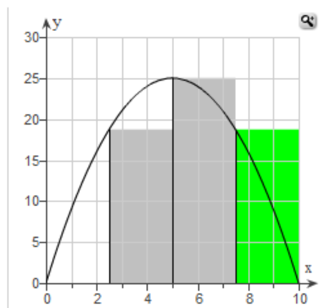
The area of this rectangle is its height times its base width Δx, or $18.75 \cdot 2.5 = 46.875$.



Repeat this process to find the height and area of the third rectangle in the subinterval [5,7.5].

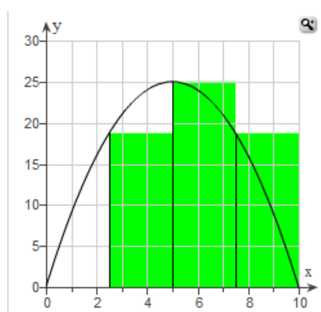
$$y = 10 \cdot 5 - 5^2$$

$$= 25$$



The height is 25, so the area is $25 \cdot 2.5 = 62.5$.

For the last subinterval, [7.5,10], the height is 18.75 and the area is 46.875. Note that this rectangle has the same area as the second rectangle.



So, the areas of the four rectangles are 0, 46.875, 62.5, and 46.875.

To compute the LRAM sum, add these areas to find the total area.

$$0 + 46.875 + 62.5 + 46.875 = 156.25$$

So, the approximate area of the region R is 156.25.

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3.

Use MRAM to estimate the area of the region enclosed between the graph of $f(x) = \frac{7}{x}$ and the x-axis for $1 \leq x \leq 89$.

To find an estimate using two rectangles, begin by partitioning the region into two equal vertical strips. The width of each rectangle is half the width of the interval [1,89].

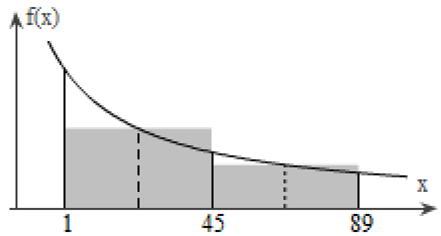
3. answer

Use MRAM to estimate the area of the region enclosed between the graph of $f(x) = \frac{7}{x}$ and the x-axis for $1 \le x \le 89$.

To find an estimate using two rectangles, begin by partitioning the region into two equal vertical strips. The width of each rectangle is half the width of the interval [1,89].

The width of each of the two rectangles is 44.

In MRAM, or the Midpoint Rectangular Approximation Method, the height of each rectangle is the value of the function at the midpoint of its base (width).



Because the midpoint of the left rectangle is at $x = 23$, the height of the left rectangle is f(23). Evaluate f(x) at $x = 23$.

$$f(23) = \frac{7}{23}$$

The area of the left rectangle is its height times its base width.

$$\text{Area} = \frac{7}{23} \cdot 44$$

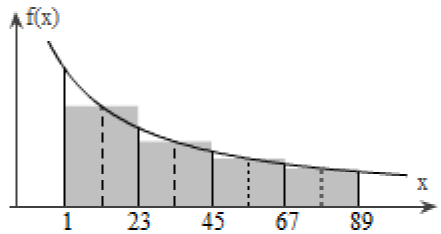
$$\approx 13.3913 \text{ (rounded to four decimal places)}$$

Likewise, the area of the right rectangle is $(44)\left(\frac{7}{67}\right) \approx 4.5970$ (rounded to four decimal places).

The two-rectangle estimate of the area using MRAM is the sum of the two areas, or 17.9883.

3. answer cont.

To approximate the area with four rectangles, divide the interval [1,89] into four equal parts that will serve as the widths of the four rectangles.



The width of each rectangle is 22.

The heights of the four rectangles are the function values at each of the four midpoints of the segments of the interval, or f(12), f(34), f(56), and f(78).

Evaluate f(x) at each of these midpoints.

$f(12) = \frac{7}{12}$ $f(34) = \frac{7}{34}$ $f(56) = \frac{7}{56}$ $f(78) = \frac{7}{78}$

$f(12) = \frac{7}{12}$ $f(34) = \frac{7}{34}$ $f(56) = \frac{7}{56}$ $f(78) = \frac{7}{78}$

Now, calculate the area of each rectangle by multiplying the height by the base width.

$$\text{Area of the first rectangle} = \frac{7}{12} \cdot 22 = \frac{154}{12}$$

$$\text{Area of the second rectangle} = \frac{7}{34} \cdot 22 = \frac{154}{34}$$

$$\text{Area of the third rectangle} = \frac{7}{56} \cdot 22 = \frac{154}{56}$$

$$\text{Area of the fourth rectangle} = \frac{7}{78} \cdot 22 = \frac{154}{78}$$

Add these four areas together to find the MRAM sum. The area estimate, rounded to four decimal places, is 22.0871.

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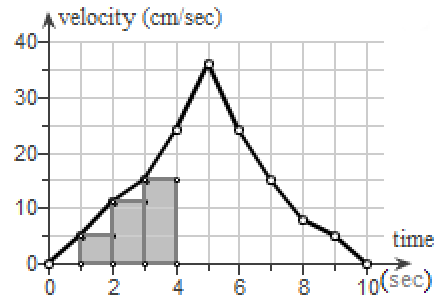
4. The table below shows the velocity of an object (cm/sec) at each sec as it moves along a track of 10 sec.

Time (sec)	0	1	2	3	4	5	6	7	8	9	10
Velocity (cm/sec)	0	5	11	15	24	36	24	15	8	5	0

Estimate the distance traveled using ten subintervals of length 1 sec with left-endpoint values.
Estimate the distance traveled using ten subintervals of length 1 sec with right-endpoint values.

To find the distance traveled using ten subintervals of length 1 sec with left-endpoint values, calculate the sum of the ten areas corresponding to the left-end samples in the graph. Notice that the first left-end rectangle has a height of 0. The next three are shown in the graph.



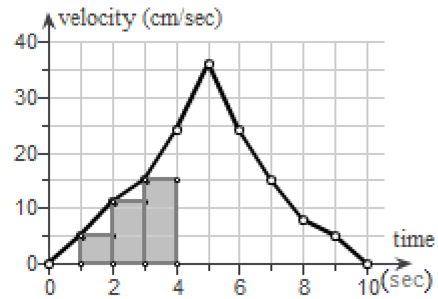
4. answer

The table below shows the velocity of an object (cm/sec) at each sec as it moves along a track of 10 sec.

Time (sec)	0	1	2	3	4	5	6	7	8	9	10
Velocity (cm/sec)	0	5	11	15	24	36	24	15	8	5	0

Estimate the distance traveled using ten subintervals of length 1 sec with left-endpoint values.
Estimate the distance traveled using ten subintervals of length 1 sec with right-endpoint values.

To find the distance traveled using ten subintervals of length 1 sec with left-endpoint values, calculate the sum of the ten areas corresponding to the left-end samples in the graph. Notice that the first left-end rectangle has a height of 0. The next three are shown in the graph.



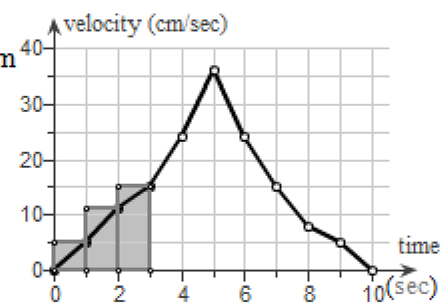
The area of the left-end rectangle from 1 to 2 is $(1)(5) = 5$.

Add the area of all 10 left-end rectangles to find the total area.

$1(0) + 1(5) + 1(11) + 1(15) + 1(24) + 1(36) + 1(24) + 1(15) + 1(8) + 1(5) = 143$

So, the estimate of the distance traveled using left-end rectangles is 143 cm.

To find the distance traveled using ten subintervals of length 1 sec with right-endpoint values, calculate the sum of the ten areas corresponding to the right-end samples in the graph. The first three are shown in the graph.



The area of the right-end rectangle from 0 to 1 is $(1)(5) = 5$.

Add the area of all 10 right-end rectangles to find the total area.

$1(5) + 1(11) + 1(15) + 1(24) + 1(36) + 1(24) + 1(15) + 1(8) + 1(5) + 1(0) = 143$

Thus, by using the right-end rectangles, the estimated distance traveled to be 143 cm.

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5. You are walking along the bank of a tidal river watching the incoming tide carry a bottle upstream. You record the velocity of the flow every 5 minutes for an hour, with the results shown in the table below.

Time (min)	0	5	10	15	20	25	30	35	40	45	50	55	60
Velocity (m/sec)	0.9	1.4	1.7	2.2	2.0	1.6	1.7	1.1	1.4	2.0	1.8	1.6	0

About how far upstream does the bottle travel during that hour? Find the **(a)** LRAM and **(b)** RRAM estimates using 12 subintervals of length 5.

(a) First convert the time to seconds.

Time (min)	Time (sec)	Time (min)	Time (sec)
0	0	35	2100
5	300	40	2400
10	600	45	2700
15	900	50	3000
20	1200	55	3300
25	1500	60	3600
30	1800		

5. answer

You are walking along the bank of a tidal river watching the incoming tide carry a bottle upstream. You record the velocity of the flow every 5 minutes for an hour, with the results shown in the table below.

Time (min)	0	5	10	15	20	25	30	35	40	45	50	55	60
Velocity (m/sec)	0.9	1.4	1.7	2.2	2.0	1.6	1.7	1.1	1.4	2.0	1.8	1.6	0

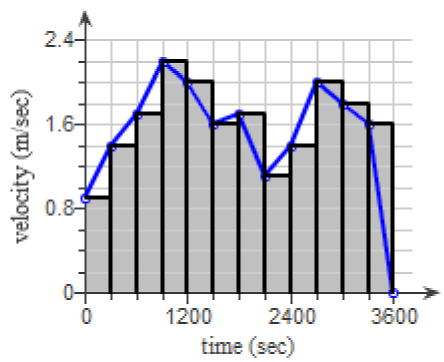
About how far upstream does the bottle travel during that hour? Find the (**a**) LRAM and (**b**) RRAM estimates using 12 subintervals of length 5.

(**a**) First convert the time to seconds.

Time (min)	Time (sec)	Time (min)	Time (sec)
0	0	35	2100
5	300	40	2400
10	600	45	2700
15	900	50	3000
20	1200	55	3300
25	1500	60	3600
30	1800		

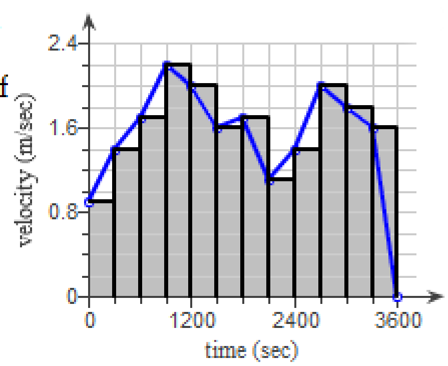
Note that the values in the table for time in the problem statement are now in seconds.

No matter which RAM approximation is being computed, add products of the form $f(x_i) \cdot \Delta x$. To find the distance traveled using twelve subintervals of length 5 min = 300 sec with left-endpoint values, calculate the sum of the twelve areas corresponding to the left-end samples in the graph.



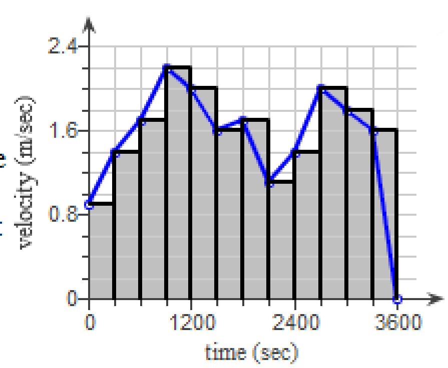
Determine the length of the base of each rectangle, Δt.

The length of the base of each rectangle is the length of each subinterval. Thus $\Delta t = 300$.



Find the area of the first left-end rectangle. Note that the subinterval for the first left-end rectangle is [0, 300].

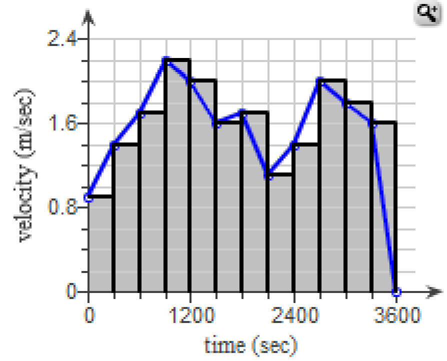
The area of a rectangle is its length times its width. The width of each rectangle is 300 and the length of the rectangle is the value of the velocity at the left endpoint of the interval. Thus, the area is $(0.9)(300) = 270$.



5. answer cont.

Determine the area of the left-end rectangle on the interval [300, 600].

The area of the left-end rectangle on the interval [300, 600] is 420.



Continuing in this manner, compute the area of the remaining 10 subintervals.

Subinterval	Area = (300)(velocity)	Subinterval	Area = 300(velocity)
[0, 300]	270	[1800, 2100]	510
[300, 600]	420	[2100, 2400]	330
[600, 900]	510	[2400, 2700]	420
[900, 1200]	660	[2700, 3000]	600
[1200, 1500]	600	[3000, 3300]	540
[1500, 1800]	480	[3300, 3600]	480

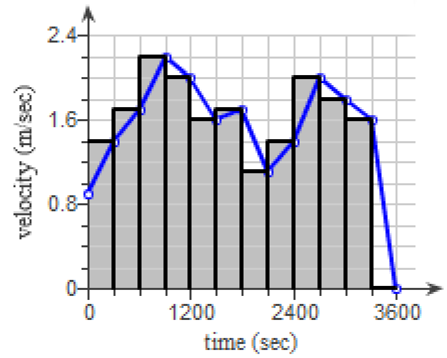
To find the LRAM estimate of the area, add the areas of all 12 subintervals.

$$270 + 420 + 510 + 660 + 600 + 480 + 510 + 330 + 420 + 600 + 540 + 480 = 5820$$

Therefore, the LRAM estimate using 12 subintervals of length 5 of the distance traveled by the bottle is 5820 m.

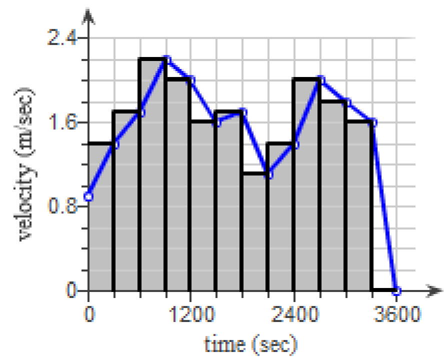
5. answer cont.

(b) To find the distance traveled using twelve subintervals of length 5 min = 300 sec with right-endpoint values, calculate the sum of the twelve areas corresponding to the right-end samples in the graph. The base of each rectangle is still $\Delta t = 300$.



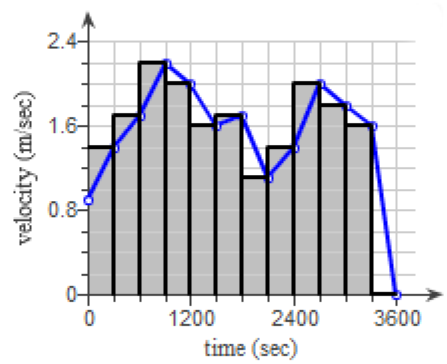
Find the area of the first right-end rectangle. Note that the subinterval for the first right-end rectangle is [0, 300].

The area of the first right-end rectangle is $(1.4)(300)$.



Determine the area of the right-end rectangle on the interval [300, 600].

The area of the right-end rectangle on the interval [300, 600] is 510.



Continuing in this manner, compute the area of the remaining 10 subintervals.

Subinterval	Area = (300)(velocity)	Subinterval	Area = 300(velocity)
[0, 300]	420	[1800, 2100]	330
[300, 600]	510	[2100, 2400]	420
[600, 900]	660	[2400, 2700]	600
[900, 1200]	600	[2700, 3000]	540
[1200, 1500]	480	[3000, 3300]	480
[1500, 1800]	510	[3300, 3600]	0

To find the RRAM estimate of the area, add the areas of all 12 subintervals.

$$420 + 510 + 660 + 600 + 480 + 510 + 330 + 420 + 600 + 540 + 480 + 0 = 5550$$

Therefore, the RRAM estimate using 12 subintervals of length 5 of the distance traveled by the bottle is 5550 m.