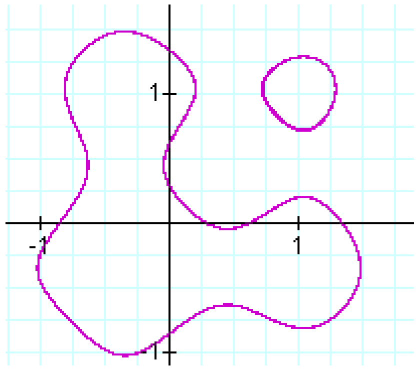
IMPLICIT DIFFERENTIATION

Consider the graph of $x^2 + y^2 + \sin 4x + \sin 4y = 1$. It's definitely not a function. However, one may want to know the slope at some point on the curve.

To find the slope one must find dy/dx.



$x^2 + y^2 + \sin 4x + \sin 4y = 1$

Here what it will look like when you have implicit differentiation mastered: (Don't worry. It will all be explained soon.)

Note: let $y' = dy/dx$.

$$\frac{d}{dx}\left[x^2 + y^2 + \sin 4x + \sin 4y\right] = \frac{d}{dx}[1]$$

$$2x + 2y \cdot y' + 4\cos 4x + (4\cos 4y)\cdot y' = 0$$

$$2y \cdot y' + (4\cos 4y)\cdot y' = -2x - 4\cos 4x$$

$$y' \cdot (2y + 4\cos 4y) = -2x - 4\cos 4x$$

$$y' = \frac{-2x - 4\cos 4x}{2y + 4\cos 4y}$$

$$y' = -\frac{x + 2\cos 4x}{y + 2\cos 4y}$$

Step 1. $\frac{d}{dx}[\text{both sides}]$.

Step 2. Isolate the y' terms.

Step 3. Factor out y'.

Step 4. Divide.

Step 5. Simplify

EXPLANATION:

Step 1. $\frac{d}{dx}[\text{both sides}]$. (Take the derivative of both sides of the equation with respect to x.) The goal is to find dy/dx so the derivative with respect to x must be taken. Take the derivative term by term. Be careful to follow the proper rules.

$$\frac{d}{dx}\left[x^2 + y^2 + \sin 4x + \sin 4y\right] = \frac{d}{dx}[1]$$

$$2x + 2y \cdot y' + 4\cos 4x + \underbrace{(4\cos 4y)\cdot y'} = 0$$

Most of them should look reasonable. Did you remember why the 4 is in on the $4\sin 4x$?
What about the terms with y in them? Why do they have y' on the end?
The answer to those questions is the same: THE CHAIN RULE.

$$\frac{d}{dx}[\sin 4y] = (4\cos 4y)\cdot y'$$

Different variables!

Step 2. Isolate the y' terms. Get all the y' terms by themselves on one side of the equation.

$$2y \cdot y' + (4\sin 4y) \cdot y' = -2x - 4\sin 4x$$

Step 3. Factor out y'.

$$y' \cdot (2y + 4\sin 4y) = -2x - 4\sin 4x$$

Step 4. Divide

$$y' = \frac{-2x - 4\cos 4x}{2y + 4\cos 4y}$$

Step 5. Simplify

$$\begin{aligned} y' &= \frac{-2x - 4\cos 4x}{2y + 4\cos 4y} \\ &= \frac{-2(x + 2\cos 4x)}{2(y + 2\cos 4y)} \\ &= -\frac{x + 2\cos 4x}{y + 2\cos 4y} \end{aligned}$$

Therefore $\dfrac{dy}{dx} = -\dfrac{x + 2\cos 4x}{y + 2\cos 4y}$

Now try to mimic the "mastered" version above. Can you do it without looking at the notes?

Example 2. Find dy/dx for $3xy + 2x^2y^3 = 4$.

$$\begin{aligned} \frac{d}{dx}\left[3xy + 2x^2y^3\right] &= \frac{d}{dx}[4] \\ (3 \cdot y + y' \cdot 3x) + (4x \cdot y^3 + 3y^2 \cdot y' \cdot 2x^2) &= 0 \\ 3y + 3x \cdot y' + 4xy^3 + 6x^2y^2 \cdot y' &= 0 \\ 3x \cdot y' + 6x^2y^2 \cdot y' &= -3y - 4xy^3 \\ y' \cdot (3x + 6x^2y^2) &= -3y - 4xy^3 \\ y' &= \frac{-3y - 4xy^3}{3x + 6x^2y^2} \end{aligned}$$

> Watch out!
> The product rule applies
>
> $$\frac{d}{dx}[3xy] = \frac{d}{dx}[3x \cdot y] = 3 \cdot y + y' \cdot 3x$$

NOTE: Do these derivatives look different? Did you notice they can have both an *x* and a *y* variable? Do you need that for graphs that are not functions?