Calculus Unit 3—Derivatives Product rule practice	Name ____________________ Date ____________________ Period ____________________

- *PROBLEM 1* : Differentiate $y = (x^3 + 7x - 1)(5x + 2)$.

- *PROBLEM 2* : Differentiate $y = x^{-2}(4 + 3x^{-3})$.

- *PROBLEM 3* : Differentiate $y = x^3 \ln x$.

- *PROBLEM 4* : Differentiate $f(x) = 6x^{\frac{3}{2}} \tan x$.

- *PROBLEM 5* : Differentiate $y = 5x^2 + \sin x \cos x$.

- *PROBLEM 6* : Differentiate $g(x) = e^x(7 - \sqrt{x})$.

SOLUTION 1 : Differentiate $y = (x^3 + 7x - 1)(5x + 2)$.

Then

$$y' = (x^3 + 7x - 1)\ D\{5x + 2\} + D\{x^3 + 7x - 1\}\ (5x + 2)$$

$$= (x^3 + 7x - 1)(5) + (3x^2 + 7)(5x + 2)$$

$$= 5x^3 + 35x - 5 + 15x^3 + 6x^2 + 35x + 14$$

$$= 20x^3 + +6x^2 + 70x + 9\ .$$

SOLUTION 2 : Differentiate $y = x^{-2}(4 + 3x^{-3})$.

Then

$$y' = x^{-2}\ D\{4 + 3x^{-3}\} + D\{x^{-2}\}\ (4 + 3x^{-3})$$

$$= x^{-2}(-9x^{-4}) + (-2x^{-3})(4 + 3x^{-3})$$

$$= -9x^{-6} - 8x^{-3} - 6x^{-6}$$

$$= -15x^{-6} - 8x^{-3}$$

$$= \frac{-15}{x^6} - \frac{8}{x^3}$$

$$= \frac{-15}{x^6} - \frac{8x^3}{x^6}$$

$$= -\ \frac{15 + 8x^3}{x^6}\ .$$

SOLUTION 3 : Differentiate $y = x^3 \ln x$.

Then

$$y' = x^3\ D\{\ln x\} + D\{x^3\}\ \ln x$$

$$= x^3\ \left(\frac{1}{x}\right) + (3x^2) \ln x$$

$$= x^2 + 3x^2 \ln x$$

$$= x^2(1 + 3 \ln x)\ .$$

SOLUTION 4 : Differentiate $f(x) = 6x^{\frac{3}{2}} \tan x$.

Then

$$f'(x) = 6x^{\frac{3}{2}} \; D\{\tan x\} + D\{6x^{\frac{3}{2}}\} \; \tan x$$

$$= 6x^{\frac{3}{2}} (\sec^2 x) + (6(\tfrac{3}{2}) x^{\frac{1}{2}}) \tan x$$

$$= 6x^{\frac{3}{2}} \sec^2 x + 9x^{\frac{1}{2}} \tan x$$

$$= 3x^{\frac{1}{2}} (2x \sec^2 x + 3 \tan x) \; .$$

SOLUTION 5 : Differentiate $y = 5x^2 + \sin x \cos x$.

Then

$$y' = 10x + \sin x \; D\{\cos x\} + D\{\sin x\} \; \cos x$$

$$= 10x + \sin x (-\sin x) + (\cos x) \cos x$$

$$= 10x + \cos^2 x - \sin^2 x$$

(An alternate answer can be given using the trigonometry identity $\cos(2x) = \cos^2 x - \sin^2 x$.)

$$= 10x + \cos(2x) \; .$$

SOLUTION 6 : Differentiate $g(x) = e^x (7 - \sqrt{x})$.

Then

$$g'(x) = e^x \; D\{7 - x^{\frac{1}{2}}\} + D\{e^x\} \; (7 - \sqrt{x})$$

$$= e^x (-\tfrac{1}{2} x^{-1/2}) + (e^x)(7 - \sqrt{x})$$

$$= \frac{-e^x}{2x^{\frac{1}{2}}} + 7e^x - \sqrt{x} e^x$$

$$= \frac{-e^x}{2x^{\frac{1}{2}}} + 7e^x \frac{2x^{\frac{1}{2}}}{2x^{\frac{1}{2}}} - \sqrt{x} e^x \frac{2x^{\frac{1}{2}}}{2x^{\frac{1}{2}}}$$

$$= \frac{-e^x + 14\sqrt{x} e^x - 2x e^x}{2\sqrt{x}}$$

$$= \frac{e^x (-1 + 14\sqrt{x} - 2x)}{2\sqrt{x}} \; .$$