Mean Value Theorem

The Mean Value Theorem is an 'existence theorem'.

> *In [mathematics](), an **existence theorem** is a theorem with a statement beginning 'there exist(s) ..', or more generally 'for all x, y, ... there exist(s) ...'.*

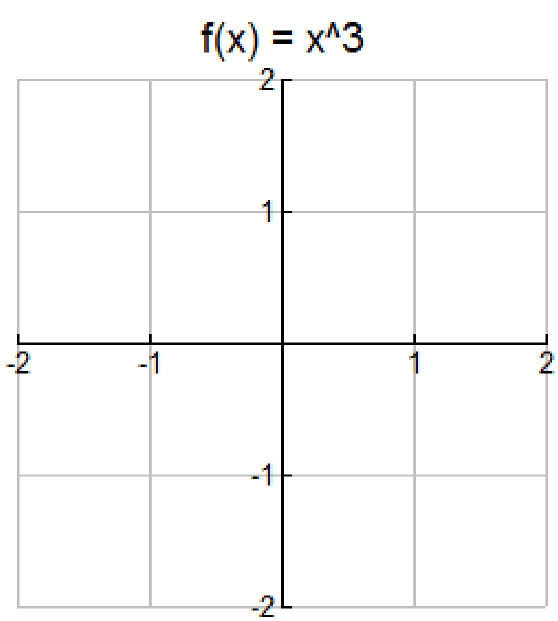
Imagine that you take a drive and average 50 *miles per hour*.

The mean value theorem guarantees that you are going exactly 50 *mph* for at least one moment during your drive. Think about it. Your average speed can't be 50 *mph* if you go slower than 50 the whole way or if you go faster than 50 the whole way. So, to average 50 *mph*, either you go exactly 50 for the whole drive, or you have to go slower than 50 for part of the drive and faster than 50 at other times. And if you're going less than 50 at one point and more than 50 at a later point (or vice versa), you have to hit exactly 50 at least once as you speed up (or slow down). You can't jump over 50 — like you're going 49 one moment then 51 the next — because speeds go up by *sliding* up the scale, not jumping. So, at some point, your speedometer slides past 50 mph, and for at least one instant, you're going exactly 50 *mph*.

That's all the mean value theorem says; there exists a place where the average rate of change is equal to the instantaneous rate of change!

Now imagine the following function: $f(x) = x^3$ for the interval $-1 \le x \le 1$

What does that look like graphically?



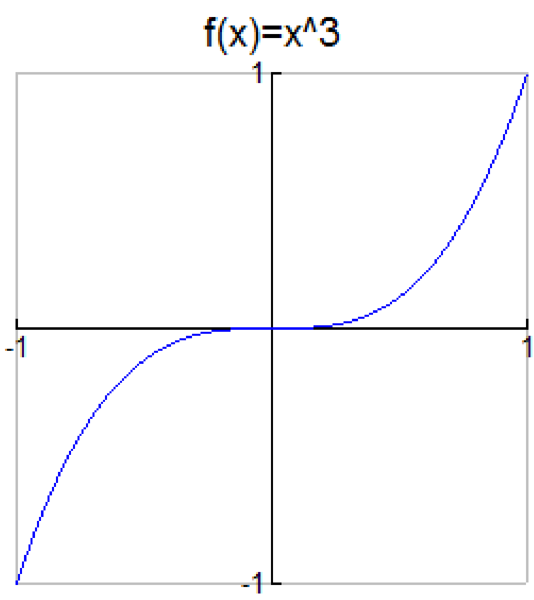
The Mean Value Theorem says:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Lets fill in the equation with what we know (in order to find the average rate of change):

Now take the derivative of the original function $f(x) = x^3$

Let's find the point where the instantaneous rate of change is equal to the average rate of change:

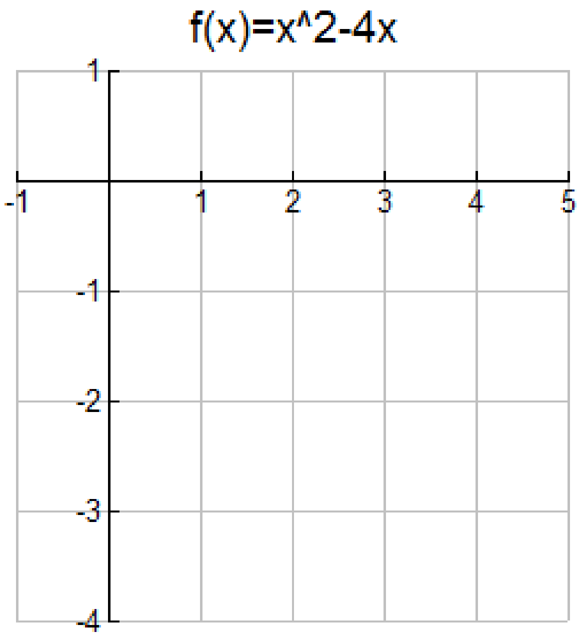


*Label the Instantaneous rate of change (tangent line)
*Label the average rate of change (secant line)

Now take a look at the following function and try it on your own:

$f(x) = x^2 - 4x$ on the interval $2 \le x \le 4$

take a look at this graphically:



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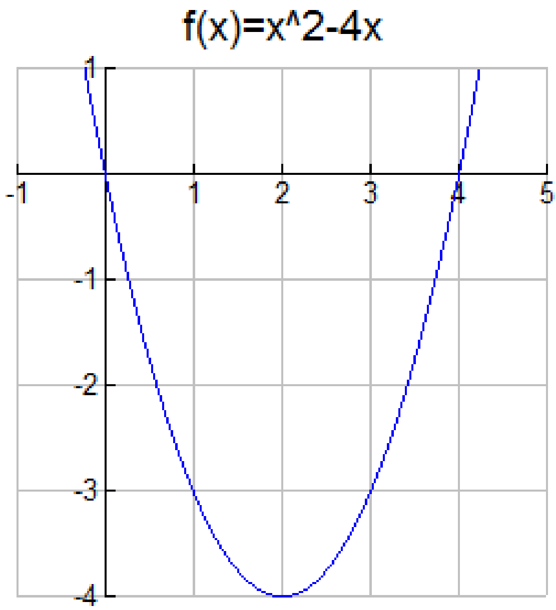
Let's find the point where the instantaneous rate of change is equal to the average rate of change:



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