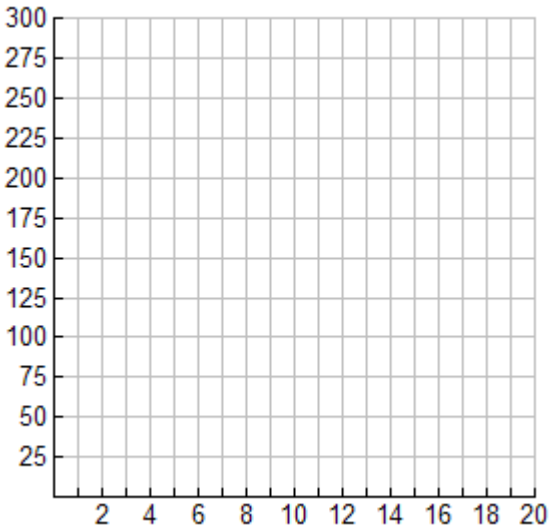
Calculus, Exploration 6.1

Consider the function $y = f(t)$ where the only things you know about the function are that

- it is a function of t
- $\frac{dy}{dt} = .05y$ (that is, the slope of $f(t)$ is 5% of the *y*-value at that point)
- $f(0) = 100$

Try to sketch a graph of $f(t)$ on the grid provided by using the following technique.

1. Predict the value for $f(1)$ using what you know about the function. (Hint: find the slope at $f(0)$.)
2. Predict another value for $f(t)$ where $t > 1$.
3. Continue to find function values for $f(t)$ by using previously calculated values.



Think:

1. What do you think is true about the concavity of the function $f(t)$?
2. Describe how accurate your sketch is compared to the actual graph of $f(t)$. Explain what differences you believe there are.
3. Describe what could be done to improve the accuracy of a sketch of $f(t)$.
4. What effect would a different "initial value" of $f(0)$ have on the graph of $f(t)$?

Expand:

1. Predict what would have happened to your sketch of $f(t)$ if you had used larger changes in *x* than you did. That is, what would have happened if you had first found $f(2)$ instead of $f(1)$ and likewise subsequently took larger steps in the *x* direction? Test your conjecture by sketching another graph of $f(t)$ on the grid provided above by using $\Delta x > 1$ (changes of *x* larger than 1).
2. Predict what would have happened to your sketch of $f(t)$ if you had used small changes in *x*?
3. Describe what needs to happen to Δx to get the "perfect" graph of $f(t)$.

Dig Deeper:

What if you didn't have any more information than $\frac{dy}{dt} = .05y$? That is, what if you didn't have an initial value? Could you describe what shape the graph of $f(t)$ must have?