- **Significance testing for two proportions**

  Visit the Hyperstat site to experience the significance test for the difference between two proportions. Be sure to only work through the 5 pages on this topic.

An experiment is conducted investigating the long-term effects of early childhood intervention programs (such as head start). In one (hypothetical) experiment, the high-school drop out rate of the experimental group (which attended the early childhood program) and the control group (which did not) were compared. In the experimental group, 73 of 85 students graduated from high school. In the control group, only 43 of 82 students graduated. Is this difference statistically significant?

1. The first step in hypothesis testing is to specify the null hypothesis and analternative hypothesis. When testing differences between proportions, the null hypothesis is that the two population proportions are equal. That is, the null hypothesis is:

   H_0: $\pi_1 = \pi_2$.
   The alternative hypothesis is: H_1: $\pi_1 \neq \pi_2$.

   In this example, the null hypothesis is:

   H_0: $\pi_{\text{intervention}} - \pi_{\text{no intervention}} = 0$.

2. The second step is to choose a significance level. Assume the 0.05 level is chosen.

3. The third step is to compute the difference between the sample proportions. In this example, $p_1 - p_2 = 73/85 - 43/82 = 0.8588 - 0.5244 = 0.3344$.

4. The fourth step is to compute p, the probability (or probability value). It is the probability of obtaining a difference between the proportions as large or larger than the difference observed in the experiment. Applying the general formula to the problem of differences between proportions

   $$z = \frac{p_1 - p_2}{s_{p_1 - p_2}}$$

   where $p_1 - p_2$ is the difference between sample proportions and

   $$s_{p_1 - p_2}$$

   is the estimated standard error of the difference between proportions. The formula for the estimated standard error is:

$$s_{p_1 - p_2} = \sqrt{\frac{p(1-p)}{n_1} + \frac{p(1-p)}{n_2}}$$

where p is a weighted average of the p_1 and p_2, n_1 is the number of subjects sampled from the first population, and n_2 is the number of subjects sampled from the second population.

4. (continued) Therefore, z = 0.344/0.0713 = 4.69. A z table can be used to find that the two-tailed probability value for a z of 4.69 is less than 0.0001.

   If $n_1 = n_2$ then p is simply the average of p_1 and p_2:

   $$p = \frac{p_1 + p_2}{2}.$$

   The two p values are averaged since the computations are based on the assumption that the null hypothesis is true. When the null hypothesis is true, both p_1 and p_2 are estimates of the same value of π. The best estimate of □ π is then the average of the two p's.

   Naturally, if one p is based on more observations than the other, it should be counted more heavily. The formula for p when $n_1 \neq n_2$is:

   $$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}.$$

   The computations for the example on early childhood intervention are:

   $$p = \frac{(85)(0.859) + (82)(0.524)}{85 + 82} = 0.695.$$

   $s_{p_1 - p_2} =$ $$s_{p_1 - p_2} = \sqrt{\frac{0.695(1 - 0.695)}{85} + \frac{0.695(1 - 0.695)}{82}} = 0.0713.$$

5. The probability computed in Step 4 is compared to the significance level stated in Step 2. Since the probability value (<0.0001) is less than the significance level of 0.05, the effect is significant.

6. Since the effect is significant, the null hypothesis is rejected. The conclusion is that the probability of graduating from high school is greater for students who have participated in the early childhood intervention program than for students who have not.
7. The results could be described in a report as:

The proportion of students from the early-intervention group who graduated from high school was 0.86 whereas the proportion from the control group who graduated was only 0.52. The difference in proportions is significant, $z = 4.69$, $p < 0.001$.

Summary of Computations

1. Compute p_1 and p_2.
2. Compute $$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}.$$
3. Compute $$s_{p_1 - p_2} = \sqrt{\frac{p(1-p)}{n_1} + \frac{p(1-p)}{n_2}}.$$
4. Compute $$z = \frac{p_1 - p_2}{s_{p_1 - p_2}}.$$
5. Use a z table to compute the probability value from z. Note that the correction for continuity is not used in the test for differences between proportions.

Assumptions

1. 1. The two proportions are independent.
2. For the normal approximation to be adequate, π should not be too close to 0 or to 1. Values between 0.10 and 0.90 allow the approximation to be adequate.
3. For the normal approximation to be adequate, there should be at least 10 subjects per group.