"We don't know a millionth of one percent about anything."
Thomas Alva Edison

## 13.1 TEST FOR GOODNESS OF FIT (Pages 702 - 716)

OVERVIEW: A chi-square test goodness of fit test is used to see if an observed sample distribution is different from a hypothesized population distribution. In other words, it is used to see if what you got is statistically different from what you expected to get.

The chi-square ($\chi^2$) statistic is calculated as follows:

$\chi^2$ = sum[(observed - expected)$^2$/expected]

Properties of $\chi^2$:

- $\chi^2$ is nonnegative in value.
- The $\chi^2$ distribution is not symmetrical. It is skewed to the right.
- All $\chi^2$ tests are 1-tail tests.
- In a goodness of fit test, the degrees of freedom is *number of categories - 1*.

# Example #1:

A die is tossed 120 times with the results displayed in the following table. Is there statistical evidence to suggest that the die is "loaded"?

| Up Face--> | **1** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|---|
| Observed frequency | 25 | 17 | 15 | 23 | 24 | 16 |
| Expected frequency | 20 | 20 | 20 | 20 | 20 | 20 |

In this situation, the degrees of freedom is 6 - 1= 5.
[Note that 5 of the categories are free to vary, but the sixth is not, since all categories have to add up to 120.]

The calculated $X^2$ is

$X^2 = (25-20)^2/20 + (17-20)^2/20 + (15-20)^2/20 + (23-20)^2/20 + (24-20)^2/20 + (16-20)^2/20 = 5.$

At the 5% level of significance, the critical region is $X^2 > 11.1$. Since our calculated $X^2$ is not in this region, we would not reject a null hypothesis that says "The die is fair."

**Alternate approach**: Using the TI-83, the P-value $\chi^2$ **cdf (**5,1E99,5) = .4158801852,which is approximately 41.6%. There is no evidence to suggest that the die is loaded.

# Example #2:

Suppose I flip a coin 100 times and get 80 heads and 20 tails.

| | Number of HEADS | Number of TAILS |
|---|---|---|
| Observed | 80 | 20 |
| Expected | 50 | 50 |

The $X^2$ statistic for this experiment is

$X^2 = (80-50)^2/50 + (20-50)^2/50 = 18 + 18 = 36.$

The degrees of freedom is 2-1 = 1.

At the 1% level of significance, the critical region for $X^2$ is $X^2 > 6.63$. Our calculated value of 36 is well into this region. There is strong evidence to suggest that the coin is not fair.

**Alternate approach:** Using the TI-83, the P-value is $X^2$ cdf (36, 1E99,1) = .00000000197. This is the probability that one would get 80 or more heads when flipping a fair coin 100times. This supports the previous statement suggesting that the coin is not fair.

The $X^2$ goodness of fit test can be used when

- All individual expected counts are at least 1.
- No more than 20% of the expected counts are less than 5.

In the examples above, these conditions were satisfied.

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