Objectives
Identify linear functions and linear equations.
Graph linear functions that represent real-world situations and give their domain and range.

Vocabulary
linear function
linear equation

Why learn this?
Linear functions can describe many real-world situations, such as distances traveled at a constant speed.

Most people believe that there is no speed limit on the German autobahn. However, many stretches have a speed limit of 120 km/h. If a car travels continuously at this speed, \( y = 120x \) gives the number of kilometers \( y \) that the car would travel in \( x \) hours. Solutions are shown in the graph.

The graph represents a function because each domain value (\( x \)-value) is paired with exactly one range value (\( y \)-value). Notice that the graph is a straight line. A function whose graph forms a straight line is called a linear function.

**EXAMPLE 1**
Identifying a Linear Function by Its Graph

Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?

A

Each domain value is paired with exactly one range value. The graph forms a line.

linear function

B

Each domain value is paired with exactly one range value. The graph is not a line.

not a linear function

C

The only domain value, 3, is paired with many different range values.

not a function

**CHECK IT OUT!**
Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?

1a.

1b.

1c.
You can sometimes identify a linear function by looking at a table or a list of ordered pairs. In a linear function, a constant change in \(x\) corresponds to a constant change in \(y\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>7</td>
</tr>
<tr>
<td>-1</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>-5</td>
</tr>
</tbody>
</table>

In this table, a constant change of \(+1\) in \(x\) corresponds to a constant change of \(-3\) in \(y\). These points satisfy a linear function. The points from this table lie on a line.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>6</td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

In this table, a constant change of \(+1\) in \(x\) does not correspond to a constant change in \(y\). These points do not satisfy a linear function. The points from this table do not lie on a line.

**Example 2**

Identifying a Linear Function by Using Ordered Pairs

Tell whether each set of ordered pairs satisfies a linear function. Explain.

**A** \(\{(2, 4), (5, 3), (8, 2), (11, 1)\}\)

Write the ordered pairs in a table. Look for a pattern.

- A constant change of \(+3\) in \(x\) corresponds to a constant change of \(-1\) in \(y\).

These points satisfy a linear function.

**B** \(\{(-10, 10), (-5, 4), (0, 2), (5, 0)\}\)

Write the ordered pairs in a table. Look for a pattern.

- A constant change of \(+5\) in \(x\) corresponds to different changes in \(y\).

These points do not satisfy a linear function.

2. Tell whether the set of ordered pairs \(\{(3, 5), (5, 4), (7, 3), (9, 2), (11, 1)\}\) satisfies a linear function. Explain.
Another way to determine whether a function is linear is to look at its equation. A function is linear if it is described by a **linear equation**. A **linear equation** is any equation that can be written in the **standard form** shown below.

### Standard Form of a Linear Equation

$$Ax + By = C$$

where $A$, $B$, and $C$ are real numbers and $A$ and $B$ are not both 0

Notice that when a linear equation is written in standard form

- $x$ and $y$ both have exponents of 1.
- $x$ and $y$ are not multiplied together.
- $x$ and $y$ do not appear in denominators, exponents, or radical signs.

<table>
<thead>
<tr>
<th>Linear</th>
<th>Not Linear</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x + 2y = 10$</td>
<td><strong>Standard form</strong></td>
</tr>
<tr>
<td>$y - 2 = 3x$</td>
<td>Can be written as $\frac{3x - y}{-2}$</td>
</tr>
<tr>
<td>$-y = 5x$</td>
<td>Can be written as $\frac{5x + y}{0}$</td>
</tr>
<tr>
<td>$3xy + x = 1$</td>
<td>$x$ and $y$ are multiplied.</td>
</tr>
<tr>
<td>$x^3 + y = -1$</td>
<td>$x$ has an exponent other than 1.</td>
</tr>
<tr>
<td>$x + \frac{6}{y} = 12$</td>
<td>$y$ is in a denominator.</td>
</tr>
</tbody>
</table>

For any two points, there is exactly one line that contains them both. This means you need only two ordered pairs to graph a line.

### Example 3

**Graphing Linear Functions**

Tell whether each function is linear. If so, graph the function.

**A**

$y = x + 3$

Write the equation in standard form.

$$-x$$

Subtraction Property of Equality

$$\frac{y - x}{3}$$

$$-x + y = 3$$

The equation is in standard form ($A = -1$, $B = 1$, $C = 3$).

The equation can be written in standard form, so the function is linear.

To graph, choose three values of $x$, and use them to generate ordered pairs. (You only need two, but graphing three points is a good check.)

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = x + 3$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$y = 0 + 3 = 3$</td>
<td>$(0, 3)$</td>
</tr>
<tr>
<td>1</td>
<td>$y = 1 + 3 = 4$</td>
<td>$(1, 4)$</td>
</tr>
<tr>
<td>2</td>
<td>$y = 2 + 3 = 5$</td>
<td>$(2, 5)$</td>
</tr>
</tbody>
</table>

**B**

$y = x^2$

This is not linear, because $x$ has an exponent other than 1.

**CAUTION**

- $y - x = y + (-x)$
- $y + (-x) = -x + y$
- $-x = -1x$
- $y = 1y$
For linear functions whose graphs are not horizontal, the domain and range are all real numbers. However, in many real-world situations, the domain and range must be restricted. For example, some quantities cannot be negative, such as time.

Sometimes domain and range are restricted even further to a set of points. For example, a quantity such as number of people can only be whole numbers. When this happens, the graph is not actually connected because every point on the line is not a solution. However, you may see these graphs shown connected to indicate that the linear pattern, or trend, continues.

**EXAMPLE 4**

**Career Application**

Sue rents a manicure station in a salon and pays the salon owner $5.50 for each manicure she gives. The amount Sue pays each day is given by $f(x) = 5.50x$, where $x$ is the number of manicures. Graph this function and give its domain and range.

Choose several values of $x$ and make a table of ordered pairs.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = 5.50x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$f(0) = 5.50(0) = 0$</td>
</tr>
<tr>
<td>1</td>
<td>$f(1) = 5.50(1) = 5.50$</td>
</tr>
<tr>
<td>2</td>
<td>$f(2) = 5.50(2) = 11.00$</td>
</tr>
<tr>
<td>3</td>
<td>$f(3) = 5.50(3) = 16.50$</td>
</tr>
<tr>
<td>4</td>
<td>$f(4) = 5.50(4) = 22.00$</td>
</tr>
<tr>
<td>5</td>
<td>$f(5) = 5.50(5) = 27.50$</td>
</tr>
</tbody>
</table>

The number of manicures must be a whole number, so the domain is $\{0, 1, 2, 3, \ldots\}$. The range is $\{0, 5.50, 11.00, 16.50, \ldots\}$.

**What if...?** At another salon, Sue can rent a station for $10.00 per day plus $3.00 per manicure. The amount she would pay each day is given by $f(x) = 3x + 10$, where $x$ is the number of manicures. Graph this function and give its domain and range.

**THINK AND DISCUSS**

1. Suppose you are given five ordered pairs that satisfy a function. When you graph them, four lie on a straight line, but the fifth does not. Is the function linear? Why or why not?

2. In Example 4, why is every point on the line not a solution?

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe how to use the information to identify a linear function. Include an example.