Objectives
Identify quadratic functions and determine whether they have a minimum or maximum.
Graph a quadratic function and give its domain and range.

Vocabulary
quadratic function
parabola
vertex
minimum
maximum

Why learn this?
The height of a soccer ball after it is kicked into the air can be described by a quadratic function. (See Exercise 51.)

The function \( y = x^2 \) is shown in the graph. Notice that the graph is not linear. This function is a quadratic function. A quadratic function is any function that can be written in the standard form \( y = ax^2 + bx + c \), where \( a \), \( b \), and \( c \) are real numbers and \( a \neq 0 \). The function \( y = x^2 \) can be written as \( y = 1x^2 + 0x + 0 \), where \( a = 1 \), \( b = 0 \), and \( c = 0 \).

In Lesson 5-1, you identified linear functions by finding that a constant change in \( x \) corresponded to a constant change in \( y \). The differences between \( y \)-values for a constant change in \( x \)-values are called first differences.

Notice that the quadratic function \( y = x^2 \) does not have constant first differences. It has constant second differences. This is true for all quadratic functions.

**Example 1**
Identifying Quadratic Functions
Tell whether each function is quadratic. Explain.

A

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>8</td>
</tr>
<tr>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

Since you are given a table of ordered pairs with a constant change in \( x \)-values, see if the second differences are constant.

Find the first differences, then find the second differences.

The function is quadratic. The second differences are constant.

B

\[ y = -3x + 20 \]

Since you are given an equation, use \( y = ax^2 + bx + c \).

This is not a quadratic function because the value of \( a \) is 0.
Tell whether each function is quadratic. Explain.

C \[ y + 3x^2 = -4 \]

Try to write the function in the form \( y = ax^2 + bx + c \) by solving for \( y \). Subtract \( 3x^2 \) from both sides.

This is a quadratic function because it can be written in the form \( y = ax^2 + bx + c \) where \( a = -3 \), \( b = 0 \), and \( c = -4 \).

**Check it Out!** Tell whether each function is quadratic. Explain.

1a. \( \{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\} \)

1b. \( y + x = 2x^2 \)

The graph of a quadratic function is a curve called a **parabola**. To graph a quadratic function, generate enough ordered pairs to see the shape of the parabola. Then connect the points with a smooth curve.

**Example 2** Graphing Quadratic Functions by Using a Table of Values

Use a table of values to graph each quadratic function.

**A** \( y = 2x^2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = 2x^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>8</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

Make a table of values. Choose values of \( x \) and use them to find values of \( y \). Graph the points. Then connect the points with a smooth curve.

**B** \( y = -2x^2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = -2x^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-8</td>
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<td>-1</td>
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Make a table of values. Choose values of \( x \) and use them to find values of \( y \). Graph the points. Then connect the points with a smooth curve.

Use a table of values to graph each quadratic function.

2a. \( y = x^2 + 2 \)

2b. \( y = -3x^2 + 1 \)

As shown in the graphs in Examples 2A and 2B, some parabolas open upward and some open downward. Notice that the only difference between the two equations is the value of \( a \). When a quadratic function is written in the form \( y = ax^2 + bx + c \), the value of \( a \) determines the direction a parabola opens.

- A parabola opens **upward** when \( a > 0 \).
- A parabola opens **downward** when \( a < 0 \).
EXAMPLE 3 Identifying the Direction of a Parabola
Tell whether the graph of each quadratic function opens upward or downward. Explain.

A  \( y = 4x^2 \)
\( a = 4 \)  \( \) Identify the value of a.
Since \( a > 0 \), the parabola opens upward.

B  \( 2x^2 + y = 5 \)
\( 2x^2 + y = 5 \)  \( -2x^2 \)  \( -2x^2 \)  \( y = -2x^2 + 5 \)  \( a = -2 \)  \( \) Identify the value of a.
Since \( a < 0 \), the parabola opens downward.

CHECK IT OUT Tell whether the graph of each quadratic function opens upward or downward. Explain.
3a. \( f(x) = -4x^2 - x + 1 \)  3b. \( y - 5x^2 = 2x - 6 \)
The highest or lowest point on a parabola is the vertex. If a parabola opens upward, the vertex is the lowest point. If a parabola opens downward, the vertex is the highest point.

Minimum and Maximum Values

WORDS  If \( a > 0 \), the parabola opens upward, and the \( y \)-value of the vertex is the minimum value of the function.
If \( a < 0 \), the parabola opens downward, and the \( y \)-value of the vertex is the maximum value of the function.

GRAPHS  \( y = x^2 + 6x + 9 \)
Vertex: \((-3, 0)\)  Minimum: 0
\( y = -x^2 + 6x - 4 \)
Vertex: \((3, 5)\)  Maximum: 5

EXAMPLE 4 Identifying the Vertex and the Minimum or Maximum
Identify the vertex of each parabola. Then give the minimum or maximum value of the function.

A  The vertex is \((1, 5)\), and the maximum is 5.

B  The vertex is \((-2, -5)\), and the minimum is -5.
Identify the vertex of each parabola. Then give the minimum or maximum value of the function.

Unless a specific domain is given, you may assume that the domain of a quadratic function is all real numbers. You can find the range of a quadratic function by looking at its graph. For the graph of \( y = x^2 - 4x + 5 \), the range begins at the minimum value of the function, where \( y = 1 \). All the \( y \)-values of the function are greater than or equal to 1. So the range is \( y \geq 1 \).

**Example 5** Finding Domain and Range

Find the domain and range.

Find the domain and range.

**Step 1** The graph opens downward, so identify the maximum.

The vertex is \((-1, 4)\), so the maximum is \(4\).

**Step 2** Find the domain and range.

D: all real numbers

R: \( y \leq 4 \)

**Think and Discuss**

1. How can you identify a quadratic function from ordered pairs? from looking at the function rule?

2. **Get Organized** Copy and complete the graphic organizer below. In each box, describe a way of identifying quadratic functions.